Lubin-Tate and Drinfeld bundles

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Abstract. Let K be a nonarchimedean local field, let h be a positive integer, and denote by D the central division algebra of invariant $1/h$ over K. The modular towers of Lubin-Tate and Drinfeld provide period rings leading to an equivalence between a category of certain $\mathrm{GL}_h(K)$-equivariant vector bundles on Drinfeld's upper half space of dimension $h-1$ and a category of certain D^*-equivariant vector bundles on the $(h-1)$-dimensional projective space.

Contents

0 Introduction

Let p be a prime number. The category of p-adic Galois representations, i.e. that of continuous representations of the absolute Galois group of a local field on finite dimensional $\mathbb{Q}_p$-vector spaces, has largely been studied through a variety of *period rings.* These play a role in J-M. Fontaine's description of p-adic Galois representations through étale (φ, Γ)-modules, as well as in the geometrically significant definition of de Rham, semi-stable and crystalline representations (cf. [20, Theorem 4.23, Chapters 5 and 6]).

In view of the p-adic Langlands program, seeking to generalize the local Langlands correspondence by matching up p-adic Galois representations with certain continuous representations of p-adic reductive groups on nonarchimedean topological vector spaces, it seems a natural question whether it is possible to also study representations of reductive groups using suitable rings of periods.

Making use of the towers of Lubin-Tate and Drinfeld – two objects from arithmetic geometry – we present a first and promising construction, showing that this novel strategy leads to very interesting results. Let us mention that the Lubin-Tate tower figures most prominently in the proof of the local Langlands correspondence in characteristic zero by Harris and Taylor [26], as well as in Strauch's purely local proof of the fact that its ℓ-adic cohomology realizes the Jacquet-Langlands correspondence (cf. [42]).

In order to describe our procedure more precisely, let K be a nonarchimedean local field of any characteristic, denote by $\mathfrak{o} = \mathfrak{o}_K$ its valuation ring, choose a uniformizer $\pi = \pi_K$ of K, and let $h \geq 1$ be an integer. Denote by $\breve{K}$ the completion of the maximal unramified extension of K, and by $\breve{\mathfrak{o}}$ its valuation ring.

For any integer $m \geq 0$ let $\mathcal{Y}_m^{(h)}$ be the generic fibre of the formal $\breve{\mathfrak{o}}$-scheme parametrizing one dimensional formal $\mathfrak{o}$-modules of height h and level structure m, constructed in [14, Section 4]. It is an étale Galois covering of the rigid analytic open unit polydisc of dimension $h-1$ over $\breve{K}$. Denoting by $D = D_K^{(h)}$ the central division algebra of invariant $1/h$ over K and by $\mathfrak{o}_D$ its valuation ring, there are commuting left actions of the groups $G_0^{(h)} := \mathrm{GL}_h(\mathfrak{o})$ and $H_0^{(h)} := \mathfrak{o}_D^\times$ on $\mathcal{Y}_m^{(h)}$ and hence on the ring $B_m^{(h)} := \mathcal{O}(\mathcal{Y}_m^{(h)})$ of its global sections.

Section 1 is concerned with computing the rings of invariants of $B_m^{(h)}$ under the actions of $G_0^{(h)}$ and $H_0^{(h)}$ (cf. Theorem 1.4 and Corollary 1.6). For any integer $m \geq 1$ set $H_m^{(h)} := 1 + \pi^m \mathfrak{o}_D$. Denoting by $\breve{K}_m$ the field obtained by adjoining the π^m-torsion points of a one dimensional Lubin-Tate formal $\mathfrak{o}$-module to $\breve{K}$, we find a $G_0^{(h)} \times H_0^{(h)}$-equivariant isomorphism $(B_m^{(h)})^{H_m^{(h)}} \simeq \breve{K}_m$.

Let $\mathcal{X}_0^{(h)} := \Omega_K^{(h)} \times_K \breve{K}$, where $\Omega_K^{(h)}$ is Drinfeld's upper half space of dimension $h-1$ over K. Interpreting $\mathcal{X}_0^{(h)}$ as the generic fibre of a formal $\breve{\mathfrak{o}}$-scheme parametrizing special formal $\mathfrak{o}_D$-modules of height h^2, Drinfeld constructed in [15, §3] a family $\mathcal{X}_m^{(h)}$ of finite étale Galois coverings of $\mathcal{X}_0^{(h)}$ with $m \geq 0$. Again, there are commuting left actions of $G_0^{(h)}$ and $H_0^{(h)}$ on $\mathcal{X}_m^{(h)}$ and hence on the ring $A_m^{(h)} := \mathcal{O}(\mathcal{X}_m^{(h)})$ of its global sections.

In Section 2, we compute the rings of invariants of $A_m^{(h)}$ under the actions of $G_0^{(h)}$ and $H_0^{(h)}$ (cf. Theorem 2.8 and Corollary 2.10). To this end, we first show that the spaces $\mathcal{X}_m^{(h)}$ are connected (cf. Theorem 2.5). Partially, this result is contained in [22, Théorème 1.1] , [18, Lemma 4.5] and [5, Theorem 2.3]), and can also be deduced from the work of P. Boyer and J-F. Dat on the cohomology of the Drinfeld tower (cf. [12], for example). We find a $G_0^{(h)} \times H_0^{(h)}$-equivariant isomorphism $(A_m^{(h)})^{G_m^{(h)}} \simeq \breve{K}_m$.

In Section 3 we combine the two modular towers of Lubin-Tate and Drinfeld, setting $\mathcal{Z}_m^{(h)} := \mathcal{X}_m^{(h)} \times_{\breve{K}_m} \mathcal{Y}_m^{(h)}$ and $C_m^{(h)} := \mathcal{O}(\mathcal{Z}_m^{(h)})$ for any integer $m \geq 0$. There are commuting left actions of the groups $G_0^{(h)}$ and $H_0^{(h)}$ on the ring $C_m^{(h)}$ whose invariants are computed in Theorem 3.2. In fact, we find equivariant isomorphisms

$$\mathcal{O}(\mathcal{Z}_m^{(h)})^{G_0^{(h)}} \simeq \mathcal{O}(\mathcal{Y}_0^{(h)}) \qquad \text{and} \qquad \mathcal{O}(\mathcal{Z}_m^{(h)})^{H_0^{(h)}} \simeq \mathcal{O}(\mathcal{X}_0^{(h)}).$$

We are interested in the following problem. For any integer $m \geq 0$ there are natural morphisms $p_m : \mathcal{Z}_m^{(h)} \to \mathcal{X}_0^{(h)}$ and $q_m : \mathcal{Z}_m^{(h)} \to \mathcal{Y}_0^{(h)}$. Given a $G_0^{(h)}$-equivariant vector bundle $\mathcal{M}$ on $\mathcal{X}_0^{(h)}$, when is there an integer $m \geq 0$ and an $H_0^{(h)}$-equivariant vector bundle $\mathcal{N}$ on $\mathcal{Y}_0^{(h)}$ together with a $G_0^{(h)} \times H_0^{(h)}$-equivariant isomorphism

$$p_m^*(\mathcal{M}) \simeq q_m^*(\mathcal{N})?$$

Due to Theorem B for quasi-Stein spaces, the category of $G_0^{(h)}$-equivariant vector bundles of finite rank on $\mathcal{X}_0^{(h)}$ is equivalent to the category of finitely generated projective $A_0^{(h)}$-modules with a semilinear action of $G_0^{(h)}$ via the global section functor (cf. Corollary A.3). We denote by $(\,\cdot\,)^\sim$ the usual quasi-inverse. Using this result, the above problem admits the following algebraic approach, familiar from the philosophy of period rings for p-adic Galois representations which we referred to above.

A $G_0^{(h)}$-equivariant vector bundle $\mathcal{M} = \tilde{M}$ of finite rank on $\mathcal{X}_0^{(h)}$ is called *Lubin-Tate* if there is an integer $m \geq 0$ such that the natural map

$$C_m^{(h)} \otimes_{B_0^{(h)}} (C_m^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}} \longrightarrow C_m^{(h)} \otimes_{A_0^{(h)}} M$$

is an isomorphism (cf. Definition 3.4). In this case,

$$\mathbb{D}_{\mathrm{LT}}(\mathcal{M}) := [(C_m^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}}]^\sim$$

turns out to be an $H_0^{(h)}$-equivariant vector bundle of finite rank on $\mathcal{Y}_0^{(h)}$ whose definition is independent of the integer $m \geq 0$ (cf. the discussion following Definition 3.4, as well as Lemma 3.5).

Likewise, an $H_0^{(h)}$-equivariant vector bundle $\mathcal{N} = \tilde{N}$ of finite rank on $\mathcal{Y}_0^{(h)}$ is called *Drinfeld* if there is an integer $m \geq 0$ such that the natural map

$$C_m^{(h)} \otimes_{A_0^{(h)}} (C_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_0^{(h)}} \longrightarrow C_m^{(h)} \otimes_{B_0^{(h)}} N$$

is an isomorphism (cf. Definition 3.4). In this case,

$$\mathbb{D}_{\mathrm{Dr}}(\mathcal{N}) := [(C_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_0^{(h)}}]^\sim$$

is a well-defined $G_0^{(h)}$-equivariant vector bundle of finite rank on $\mathcal{X}_0^{(h)}$.

It is a formality to show that the functors $\mathbb{D}_{\mathrm{LT}}$ and $\mathbb{D}_{\mathrm{Dr}}$ are mutually quasi-inverse equivalences between the categories of Lubin-Tate and Drinfeld bundles on $\mathcal{X}_0^{(h)}$ and $\mathcal{Y}_0^{(h)}$, respectively (cf. Theorem 3.7). The nontrivial part of the theory is rather concerned with the construction of interesting examples. Using

Galois descent, we show that if V and W are finite dimensional smooth representations of $G_0^{(h)}$ and $H_0^{(h)}$ over $\breve{K}$, respectively, then the equivariant vector bundles

$$\mathcal{M}(V) := \mathcal{O}_{\mathcal{X}_0^{(h)}} \otimes_{\breve{K}} V \qquad \text{and} \qquad \mathcal{N}(W) := \mathcal{O}_{\mathcal{Y}_0^{(h)}} \otimes_{\breve{K}} W$$

are Lubin-Tate and Drinfeld, respectively (cf. Theorem 3.8). Other examples are provided by the structure sheaves of the coverings $\mathcal{X}_m^{(h)}$ and $\mathcal{Y}_m^{(h)}$, respectively (cf. Remark 3.9).

In Lemma 3.10 we show that the ring $A_m^{(h)}$ (resp. $B_m^{(h)}$) is $(\breve{K}_m, G_m^{(h)})$-regular (resp. $(\breve{K}_m, H_m^{(h)})$-regular) in the sense of [20, Definition 2.8]. In Lemma 3.11 we then give an alternative characterization for equivariant vector bundles to be Lubin-Tate or Drinfeld. As a consequence, the categories of Lubin-Tate and Drinfeld bundles enjoy many good formal properties (cf. Theorem 3.12).

In order to study objects which are equivariant under the full groups $G^{(h)} := \mathrm{GL}_h(K)$ and $H^{(h)} := D^*$, we consider the Rapoport-Zink spaces $\underline{\mathcal{X}}_m^{(h)}$ and $\underline{\mathcal{Y}}_m^{(h)}$ of the moduli problems of Drinfeld and Lubin-Tate, as well as the corresponding period spaces $\mathcal{X}_0^{(h)}$ and $\mathbb{P}_{\breve{K}}^{h-1}$ (cf. Section 4). The latter carry actions of $G^{(h)}$ and $H^{(h)}$, respectively, and the notions of equivariant Lubin-Tate and Drinfeld bundles are generalized in Definition 4.2.

We follow Fargue's exposition in [19, Chapitre I, Section IV.11], to define an equivalence between the category of $H^{(h)}$-equivariant coherent modules on $\mathbb{P}_{\breve{K}}^{h-1}$ and the category of so-called $H^{(h)}$-equivariant cartesian coherent modules on the Lubin-Tate tower (cf. Theorem 4.1). We use this result to define two mutually quasi-inverse functors, again denoted by $\mathbb{D}_{\mathrm{LT}}$ and $\mathbb{D}_{\mathrm{Dr}}$, between the category of $G^{(h)}$-equivariant Lubin-Tate bundles on $\mathcal{X}_0^{(h)}$ and the category of $H^{(h)}$-equivariant Drinfeld bundles on $\mathbb{P}_{\breve{K}}^{h-1}$ (cf. Theorem 4.4). The latter contains the category of all finite dimensional smooth representations of $H^{(h)}$ over $\breve{K}$ as a full subcategory (cf. Theorem 4.5).

We closely examine the abelian case of height one (cf. Proposition 4.6) and deduce that the above correspondence satisfies a general compatibility relation on traces (cf. Theorem 4.7). This raises the question of how it is related to the Jacquet-Langlands correspondence (cf. Remark 4.8).

The above results rely on a natural functoriality property underlying the moduli problems of Sections 1 and 2 (cf. Section 5). If $L|K$ is a finite field extension of degree n and ramification index e, we recall how to obtain $\mathrm{GL}_h(\mathfrak{o}_L) \times \mathfrak{o}_{D_L^{(h)}}^{\times}$-equivariant morphisms

$$\mathrm{i}_{\mathrm{L|K}} : \mathcal{X}_{em,L}^{(h)} \longrightarrow \mathcal{X}_{m,K}^{(nh)} \qquad \text{and} \qquad \mathrm{r}_{\mathrm{L|K}} : \mathcal{Y}_{em,L}^{(h)} \longrightarrow \mathcal{Y}_{m,K}^{(nh)}$$

for any integer $m \geq 0$, satisfying certain natural conditions (cf. Proposition 5.1). If the equivariant objects under consideration arise from finite dimensional smooth representations, the pullback functors $\mathrm{i}_{\mathrm{L|K}}^*$ and $\mathrm{r}_{\mathrm{L|K}}^*$ respect the properties of being Lubin-Tate and Drinfeld, respectively, and commute nicely with the functors $\mathbb{D}_{\mathrm{LT}}$ and $\mathbb{D}_{\mathrm{Dr}}$ (cf. Theorems 5.2 and 5.3).

Let us point out that also L. Fargues, building on ideas of G. Faltings, constructed a correspondence between certain smooth equivariant objects on the period spaces associated with the deformation spaces of Lubin-Tate and Drinfeld (cf. [19, Chapitre I, Théorème IV.13.1]). His correspondence is even an equivalence of topoi and is a formal consequence of the construction of an equivariant isomorphism between the two towers. On the other hand, it does not seem to apply to coherent module sheaves and is by far more complicated than our explicit and elementary approach.

Finally, many of our methods and arguments are general enough to hope for similar functorial correspondences involving other p-adic period domains and thus other p-adic reductive groups.

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Conventions and notation. Let K denote a nonarchimedean local field, i.e. a field which is locally compact with respect to the topology defined by a nonarchimedean nontrivial normalized valuation v_K. Let $\mathfrak{o}$ and k denote the valuation ring and the residue class field of K, respectively, and let q be the cardinality of k. We choose a uniformizer $\pi = \pi_K$ of K and a separable closure k^s of k. Let $\breve{K} := \widehat{K^{\mathrm{nr}}}$ denote the completion of the maximal unramified extension of K, and let $\breve{\mathfrak{o}}$ denote the valuation ring of $\breve{K}$.
If $h \geq 1$ is an integer we denote by $D = D_K^{(h)}$ the central division algebra of invariant $1/h$ over K. Let $\mathrm{Nrd} : D \to K$ denote the reduced norm of D over K. The valuation v_K extends to a valuation $v_D := v_K \circ \mathrm{Nrd}$ on D, and we denote by $\mathfrak{o}_D$ the corresponding valuation ring of D. We set $G^{(h)} := \mathrm{GL}_h(K)$ and $G_0^{(h)} := \mathrm{GL}_h(\mathfrak{o})$, as well as $H^{(h)} := D^*$ and $H_0^{(h)} := \mathfrak{o}_D^\times$. If R is a ring we denote by $\mathrm{M}_h(R)$ the ring of $(h \times h)$-matrices over R. For any integer $m \geq 1$ we let $G_m^{(h)} := 1 + \pi^m \mathrm{M}_h(\mathfrak{o})$ and $H_m^{(h)} := 1 + \pi^m \mathfrak{o}_D$ denote the principal congruence subgroups of $G^{(h)}$ and $H^{(h)}$ of level π^m, respectively.

1 Invariants in the Lubin-Tate tower

For Drinfeld's theory of formal $\mathfrak{o}$-modules with level structure we refer to [14, §4].

Let $\mathcal{C}$ be the category of commutative unital complete noetherian local $\breve{\mathfrak{o}}$-algebras $R = (R, \mathfrak{m}_R)$ with residue field $R/\mathfrak{m}_R \simeq k^s$. If R is an object of $\mathcal{C}$, if H is a one dimensional formal $\mathfrak{o}$-module over R, and if $\alpha \in \mathfrak{o}$, then we denote by $[\alpha]_H = [\alpha]_H(X) \in R[[X]]$ the corresponding endomorphism of H. Recall from [23, Lemma 4.1], that either the power series $[\pi]_H$ reduces to zero modulo the ideal $\mathfrak{m}_R R[[X]]$ or else there is a uniquely determined positive integer h and a power series $f \in k^s[[X]]$ with

$$[\pi]_H(X) \bmod \mathfrak{m}_R = f(X^{q^h}) \text{ and } f'(0) \neq 0.$$

In the latter case, the integer h is called the *height* of the formal $\mathfrak{o}$-module H.

We fix an integer $h \geq 1$ and a one dimensional formal $\mathfrak{o}$-module $\mathbb{H}^{(h)}$ of height h over k^s which is defined over k. Up to $\mathfrak{o}$-linear isomorphism (defined over k^s) there is exactly one such module, and we have

$$\mathrm{End}_{\mathfrak{o}}(\mathbb{H}^{(h)}) \simeq \mathfrak{o}_D, \tag{1}$$

where $\mathfrak{o}_D$ is the valuation ring of the central division algebra $D = D_K^{(h)}$ of invariant $1/h$ over K (cf. [14, Propositions 1.6 and 1.7]).

For any integer $m \geq 0$ consider the set valued functor $\mathfrak{Y}_m^{(h)}$ on $\mathcal{C}$ which associates to an object R of $\mathcal{C}$ the set of isomorphism classes $[(H, \rho, \varphi)]$ of triples (H, ρ, φ), where H is a one dimensional formal $\mathfrak{o}$-module of height h over R, ρ is an $\mathfrak{o}$-linear isomorphism

$$\rho : \mathbb{H}^{(h)} \longrightarrow H \bmod \mathfrak{m}_R,$$

and $\varphi : (\pi^{-m}\mathfrak{o}/\mathfrak{o})^h \to (\mathfrak{m}_R, +_H)$ is a homomorphism of abstract $\mathfrak{o}$-modules such that the power series $\prod_{\alpha \in (\pi^{-m}\mathfrak{o}/\mathfrak{o})^h}(X - \varphi(\alpha))$ divides $[\pi^m]_H(X)$ in $R[[X]]$.

If m' and m are integers with $m' \geq m \geq 0$ then we define a natural transformation

$$\mathfrak{Y}_{m'}^{(h)} \to \mathfrak{Y}_m^{(h)} \tag{2}$$

by sending the isomorphism class of a triple (H, ρ, φ) defined over an object R of $\mathcal{C}$ to the class of the triple $(H, \rho, \varphi|_{(\pi^{-m}\mathfrak{o}/\mathfrak{o})^h})$ via the $\mathfrak{o}$-linear embedding $(\pi^{-m}\mathfrak{o}/\mathfrak{o})^h \subseteq (\pi^{-m'}\mathfrak{o}/\mathfrak{o})^h$.

The following fundamental theorem is due to Drinfeld (cf. [14, Propositions 4.2 and 4.3]). In the case $m = 0$ and $\mathfrak{o} = \mathbb{Z}_p$ it was first proved by Lubin and Tate, building upon the work of Lazard (cf. [31, Theorem 3.1]). If $m = 0$ and if $\mathfrak{o}$ is arbitrary, a concise proof can be found in [43].

Theorem 1.1 (Lubin-Tate, Drinfeld)**.** *Let $h \geq 1$ be an integer.*

(i) *For any integer $m \geq 0$ the functor $\mathfrak{Y}_m^{(h)}$ is representable by an object $R_m^{(h)}$ of $\mathcal{C}$. The local ring $R_m^{(h)}$ is regular.*

(ii) *If m and m' are integers with $m' \geq m \geq 0$ then the homomorphism of local rings $R_m^{(h)} \to R_{m'}^{(h)}$ induced by* (2) *is finite and flat.*

(iii) *The ring $R_0^{(h)}$ is noncanonically isomorphic to the ring $\breve{\mathfrak{o}}[[t_1, \ldots, t_{h-1}]]$ of formal power series in $h-1$ indeterminates over $\breve{\mathfrak{o}}$.* $\square$

For any integer $m \geq 0$ there are commuting left actions of $G_0^{(h)}$ and $H_0^{(h)}$ on the functor $\mathfrak{Y}_m^{(h)}$ for which the morphisms (2) are equivariant. They are given by

$$(g, d) \cdot [(H, \rho, \varphi)] := [(H, \rho \circ d^{-1}, \varphi \circ g^{-1})] \quad \text{for} \quad d \in H_0^{(h)} \text{ and } g \in G_0^{(h)}, \tag{3}$$

where the action of $H_0^{(h)}$ makes use of the identification in (1). The action of the subgroup $\{(\alpha, \alpha) \mid \alpha \in \mathfrak{o}^\times\}$ of $G_0^{(h)} \times H_0^{(h)}$ on the functor $\mathfrak{Y}_m^{(h)}$ is trivial.

For any integer $m \geq 0$ we let $\mathcal{Y}_m^{(h)} := (\mathfrak{Y}_m^{(h)})^{\mathrm{rig}}$ be the rigid analytic $\breve{K}$-variety associated with the formal $\breve{\mathfrak{o}}$-scheme $\mathfrak{Y}_m^{(h)} = \mathrm{Spf}(R_m^{(h)})$ (cf. [28, Section 7]). Let

$$B_m^{(h)} := \mathcal{O}(\mathcal{Y}_m^{(h)})$$

be the $\breve{K}$-algebra of global rigid analytic functions on $\mathcal{Y}_m^{(h)}$. By functoriality, $\mathcal{Y}_m^{(h)}$ and $B_m^{(h)}$ carry commuting left actions of $G_0^{(h)}$ and $H_0^{(h)}$, respectively.

Theorem 1.2. *Let $h \geq 1$ be an integer.*

(i) *For any integer $m \geq 0$ the rigid analytic $\breve{K}$-variety $\mathcal{Y}_m^{(h)}$ is smooth connected and quasi-Stein. The $\breve{K}$-algebra $B_m^{(h)}$ is an integrally closed integral domain.*

(ii) *If m' and m are integers with $m' \geq m \geq 0$ then the morphism*

$$\mathcal{Y}_{m'}^{(h)} \longrightarrow \mathcal{Y}_m^{(h)} \tag{4}$$

induced by (2) is a finite étale Galois covering with Galois group $G_m^{(h)}/G_{m'}^{(h)}$.

(iii) *The space $\mathcal{Y}_0^{(h)}$ is noncanonically isomorphic to the rigid analytic open unit polydisc $\mathring{\mathbb{B}}_{\breve{K}}^{h-1}$ of dimension $h-1$ over $\breve{K}$.*

Proof: All assertions follow from Theorem 1.1 and the properties of the rigidification functor (cf. [28, Section 7]). That $B_m^{(h)}$ is an integrally closed integral domain follows from Lemma A.1. $\square$

Remark 1.3. It is a classical result that all formal $\mathfrak{o}$-modules of height one over $\breve{\mathfrak{o}}$ are isomorphic (cf. [30, Lemma 2]). In this case $R_m^{(1)} = \breve{\mathfrak{o}}_m$ is the valuation ring of the finite Galois extension $B_m^{(1)} = \breve{K}_m$ of $\breve{K}$ obtained by adjoining the π^m-torsion points of any Lubin-Tate formal $\mathfrak{o}$-module of height one over $\mathfrak{o}$ to $\breve{K}$ (cf. [30, Theorem 3]). We have $\mathrm{Gal}(\breve{K}_m|\breve{K}) \simeq (\mathfrak{o}/\pi^m\mathfrak{o})^\times \simeq G_0^{(1)}/G_m^{(1)}$ for any integer $m \geq 0$.

The following result heavily relies on the work [41] of Strauch.

Theorem 1.4. *Let $h \geq 1$ and $m \geq 0$ be integers.*

(i) *We have $(B_m^{(h)})^{G_0^{(h)}} = B_0^{(h)}$.*

(ii) *We have $(B_m^{(h)})^{H_m^{(h)}} = \breve{K}_m$. Viewing $\breve{K}_m$ as a left $\mathfrak{o}^\times$-module via the homomorphism $\mathfrak{o}^\times \to (\mathfrak{o}/\pi^m\mathfrak{o})^\times \simeq \mathrm{Gal}(\breve{K}_m|\breve{K})$ (cf. Remark 1.3), the induced left actions of $G_0^{(h)}$ and $H_0^{(h)}$ on $\breve{K}_m$ are given by $g \cdot \alpha = \det(g)^{-1}(\alpha)$ and $\delta \cdot \alpha = \mathrm{Nrd}(\delta)(\alpha)$ for all elements $g \in G_0^{(h)}$, $\delta \in H_0^{(h)}$ and $\alpha \in \breve{K}_m$.*

(iii) *If N is a finitely generated projective $B_m^{(h)}$-module with a semilinear action of $G_0^{(h)}/G_m^{(h)}$, then the natural map $B_m^{(h)} \otimes_{B_0^{(h)}} N^{G_0^{(h)}} \to N$ is an isomorphism.*

Proof: Assertion (i) is a direct consequence of Theorem 1.2. As for (ii) we start with the following lemma, built upon a result of Gross and Hopkins (cf. [23, Proposition 14.18]).

Lemma 1.5. *We have $(B_0^{(h)})^H = \breve{K}$ for any open subgroup H of $H_0^{(h)}$.*

Proof: If $H = H_0^{(h)}$ this follows as in [23, Proposition 14.18]. In the general case we may assume H to be normal in $H_0^{(h)}$, so that $(B_0^{(h)})^H$ is a finite Galois extension of the field $\breve{K}$ (note that $(B_0^{(h)})^H \subseteq B_0^{(h)}$ is an integral domain by Theorem 1.2). However, $\breve{K}$ is algebraically closed in $B_0^{(h)} \subseteq \breve{K}[[t_1, \ldots, t_{h-1}]]$. □

By [41, Corollary 3.4 and Theorem 4.4], there is an equivariant embedding $\breve{K}_m \subseteq B_m^{(h)}$ with the actions of $G_0^{(h)}$ and $H_0^{(h)}$ on $\breve{K}_m$ as given above. Note that the actions in [41] are from the *right* and are related to our actions by taking inverses. Thus, $\breve{K}_m \subseteq (B_m^{(h)})^{H_m^{(h)}}$.

On the other hand, since the actions of $G_0^{(h)}$ and $H_0^{(h)}$ on $B_m^{(h)}$ commute, the ring $(B_m^{(h)})^{H_m^{(h)}}$, which is an integral domain by Theorem 1.2, is finite over $(B_0^{(h)})^{H_m^{(h)}} = \breve{K}$ (cf. Lemma 1.5). Thus, $(B_m^{(h)})^{H_m^{(h)}}$ is a field and a finite Galois extension of $\breve{K}_m$ in $B_m^{(h)}$. It follows from [41, Proposition 4.2], that $\breve{K}_m$ is separably closed in $B_m^{(h)}$. Indeed, if $E|\breve{K}_m$ is a finite separable extension inside $B_m^{(h)}$ then its valuation ring $\mathfrak{o}_E$ is contained in $R_m^{(h)}$ (cf. [28, Theorem 7.4.1]). Let π_m and π_E be uniformizers of $\breve{K}_m$ and E, respectively. There is an integer $e \geq 1$ such that $\pi_m \mathfrak{o}_E = \pi_E^e \mathfrak{o}_E$. By [41, Proposition 4.2], the ring $R_m^{(h)}/\pi_m R_m^{(h)}$ is reduced, so that $e = 1$. Thus, the extension $\mathfrak{o}_E|\mathfrak{o}_{\breve{K}_m}$ is étale. Since $\mathfrak{o}_{\breve{K}_m}$ is strictly henselian we must have $\mathfrak{o}_E = \mathfrak{o}_{\breve{K}_m}$ and thus $E = \breve{K}_m$. Therefore, $(B_m^{(h)})^{H_m^{(h)}} = \breve{K}_m$.

Finally, assertion (iii) follows from Corollary A.3 and Theorem A.4. □

Corollary 1.6. *Let $m \geq 0$ be an integer. We have $(B_m^{(h)})^H = (\breve{K}_m)^{\mathrm{Nrd}(H)}$ for any open subgroup H of $H_0^{(h)}$.*

Proof: As above, the ring $(B_m^{(h)})^H$ is a finite Galois extension of $(B_0^{(h)})^H = \breve{K}$ inside $B_m^{(h)}$. According to the proof of Theorem 1.4 we have $(B_m^{(h)})^H \subseteq \breve{K}_m$. Moreover, the field $\breve{K}_m$ is stable under the action of $H_0^{(h)}$ which factors through the reduced norm. Therefore, $(\breve{K}_m)^{\mathrm{Nrd}(H)} = (\breve{K}_m)^H \subseteq (B_m^{(h)})^H \subseteq (\breve{K}_m)^H$. □

2 Invariants in the Drinfeld tower

For Drinfeld's theory of special formal $\mathfrak{o}_D$-modules we refer to [4], [15] and [21].

We fix an h-dimensional special formal $\mathfrak{o}_D$-module $\mathbb{G}^{(h)}$ of height h^2 over k^s which is defined over k. The $\mathfrak{o}_D$-module $\mathbb{G}^{(h)}$ is unique up to isogeny, and there is an isomorphism

(5) $$\mathrm{End}_{\mathfrak{o}_D}(\mathbb{G}^{(h)}) \otimes_{\mathfrak{o}} K \simeq \mathrm{M}_h(K)$$

of K-algebras (cf. [4, Propositions II.5.2 and II.5.3] for the case $h = 2$).

Let $\mathrm{Nilp}_{\mathfrak{o}}$ denote the category of commutative unital $\mathfrak{o}$-algebras in which the image of π is nilpotent. Define the set valued functor $\mathfrak{X}_0^{(h)}$ on $\mathrm{Nilp}_{\mathfrak{o}}$ by associating with an object R of $\mathrm{Nilp}_{\mathfrak{o}}$ the set of isomorphism classes $[(\psi, G, \rho)]$ of triples (ψ, G, ρ), where $\psi : k^s \to R/\pi R$ is a homomorphism of $\mathfrak{o}$-algebras, G is a special formal $\mathfrak{o}_D$-module of height h^2 over R and $\rho : \psi_*(\mathbb{G}^{(h)}) \to G \bmod \pi$ is an $\mathfrak{o}_D$-equivariant quasi-isogeny of height zero (cf. [15, §2.A] or [4, Section II.7.1] for the notion of a quasi-isogeny and of its height).

There are commuting left actions of the groups $G_0^{(h)} = \mathrm{GL}_h(\mathfrak{o})$ and $H_0^{(h)} = \mathfrak{o}_D^\times$ on the functor $\mathfrak{X}_0^{(h)}$, given as follows. Any element $g \in G_0^{(h)}$ defines a quasi-isogeny of $\mathbb{G}^{(h)}$ of height zero via (5), and we set

$$g \cdot [(\psi, G, \rho)] := [(\psi, G, \rho \circ \psi_*({}^t g))],$$

where ${}^t g$ denotes the transpose of g. We emphasize that this definition of the $G_0^{(h)}$-action on $\mathfrak{X}_0^{(h)}$ differs from the usual one by the automorphism $(g \mapsto {}^t g^{-1})$ of the group $G^{(h)} = \mathrm{GL}_h(K)$.

Given a special formal $\mathfrak{o}_D$-module and an element $\delta \in H_0^{(h)}$, we let ${}^\delta G$ be the special formal $\mathfrak{o}_D$-module obtained by pulling back the action of $\mathfrak{o}_D$ via conjugation by δ. In this way, the action of δ^{-1} on G defines an $\mathfrak{o}_D$-equivariant quasi-isogeny $\delta^{-1} : G \to {}^\delta G$ of height zero, and we set

$$\delta \cdot [(\psi, G, \rho)] := [(\psi, {}^\delta G, \rho \circ \psi_*(\delta^{-1}))].$$

Clearly, this action of $H_0^{(h)}$ on $\mathfrak{X}_0^{(h)}$ is trivial. Further, the action of the subgroup $\{(\alpha, \alpha) \mid \alpha \in \mathfrak{o}^\times\}$ of $G_0^{(h)} \times H_0^{(h)}$ on the functor $\mathfrak{X}_0^{(h)}$ is trivial.

Let $\Omega_K^{(h)}$ denote Drinfeld's upper half space of dimension $h-1$ over K, obtained by removing all K-rational hyperplanes from $\mathbb{P}_K^{h-1}$ (cf. [37, §1]). Set $\mathcal{X}_0^{(h)} := \Omega_K^{(h)} \times_K \breve{K}$. There is a natural action of the group $G^{(h)} = \mathrm{GL}_h(K)$ on the space $\Omega_K^{(h)}$ which we change via the automorphism $(g \mapsto {}^t g^{-1})$ of the group $G^{(h)}$. It extends to an action on $\mathcal{X}_0^{(h)}$ over $\breve{K}$. We emphasize that we let $G^{(h)}$ act trivially on $\breve{K}$. Further, let the group $H^{(h)} = D^*$ act trivially on $\mathcal{X}_0^{(h)}$.

The following fundamental theorem is due to Drinfeld (cf. [15, Theorem 2.A]; see also [4, Théorèmes II.8.4, II.9.3 and II.9.5], as well as [21, Chapitre III, Théorème 3.1.1]).

Theorem 2.1 (Drinfeld)**.** *Let $h \geq 1$ be an integer. The functor $\mathfrak{X}_0^{(h)}$ is pro-representable by a formal $\breve{\mathfrak{o}}$-scheme which is locally formally of finite type. Its generic fibre $(\mathfrak{X}_0^{(h)})^{rig}$ is $G_0^{(h)} \times H_0^{(h)}$-equivariantly isomorphic to the rigid analytic $\breve{K}$-space $\mathcal{X}_0^{(h)}$.* □

According to Theorem 2.1 there is a universal special formal $\mathfrak{o}_D$-module over the formal $\breve{\mathfrak{o}}$-scheme $\mathfrak{X}_0^{(h)}$ which may be used to define a certain family of rigid analytic $\breve{K}$-spaces $(\mathcal{X}_m^{(h)})_{m \geq 0}$ (cf. [15, §3], where these spaces are denoted $\Sigma^{h,m}$; see also [4, Section II.13], and [21, Section IV.1]). Each of the spaces $\mathcal{X}_m^{(h)}$

carries commuting left actions of the groups $G_0^{(h)}$ and $H_0^{(h)}$, and if m' and m are integers with $m' \geq m \geq 0$ then there are equivariant morphisms

$$\mathcal{X}_{m'}^{(h)} \longrightarrow \mathcal{X}_m^{(h)} . \tag{6}$$

The following results are all implicit in the construction of the spaces $\mathcal{X}_m^{(h)}$ or follow from [37, §1 Proposition 4].

Theorem 2.2. *Let $h \geq 1$ be an integer. For any integer $m \geq 0$ the rigid analytic $\breve{K}$-variety $\mathcal{X}_m^{(h)}$ is smooth and quasi-Stein. If m' and m are integers with $m' \geq m \geq 0$ then the morphism (6) is finite étale and Galois with Galois group $H_m^{(h)}/H_{m'}^{(h)}$.* □

Remark 2.3. If $h = 1$ then there are isomorphisms $\mathcal{X}_m^{(1)} \simeq \mathrm{Sp}(\breve{K}_m)$ for all integers $m \geq 0$ with $\breve{K}_m$ as in Remark 1.3. The field $\breve{K}_m$ is a left $\mathfrak{o}^\times$-module via the homomorphism $\mathfrak{o}^\times \to (\mathfrak{o}/\pi^m\mathfrak{o})^\times \simeq \mathrm{Gal}(\breve{K}_m|\breve{K})$. The resulting left actions of $G_0^{(1)} = \mathfrak{o}^\times$ and $H_0^{(1)} = \mathfrak{o}^\times$ on $\breve{K}_m$, obtained by transport of structure, are given by $g \cdot \alpha = g^{-1}(\alpha)$ and $\delta \cdot \alpha = \delta(\alpha)$ for all elements $g \in G_0^{(1)}$, $\delta \in H_0^{(1)}$ and $\alpha \in \breve{K}_m$.

As a supplement to the results [22, Théorème 1.1], [18, Lemma 4.5], and [5, Theorem 2.3], concerning the connected components of the spaces $\mathcal{X}_m^{(h)}$, we shall prove the following two theorems.

Theorem 2.4. *Let $h \geq 1$ be an integer. The rigid analytic K-variety $\Omega_K^{(h)}$ is smooth and geometrically connected. In particular, the ring of global sections of $\Omega_K^{(h)} \times_K F$ is an integrally closed integral domain for any complete valued field extension F of K.*

Proof: By [37, §1 Proposition 1], the space $\Omega_K^{(h)}$ is an admissible open subset of $\mathbb{P}_K^{h-1}$. Therefore, $\Omega_K^{(h)} \times_K F$ is smooth over F.

Consider the admissible covering $(\Omega_n^{(h)})_{n\geq 1}$ of $\Omega_K^{(h)}$, constructed in the proof of [37, §1 Proposition 1], consisting of an increasing sequence of admissible open subsets. According to the proof of [37, §1 Proposition 6], each $\Omega_n^{(h)}$ admits an admissible covering by open subsets $(V_{i,n})_i$ with pairwise non-empty intersections such that each $V_{i,n}$ is isomorphic to the product of an open polydisc with a closed polydisc. Thus, each space $V_{i,n}$ is geometrically connected, and we may use [10, p. 492] to conclude that so is $\Omega_K^{(h)}$.

The last assertion of the theorem follows from Lemma A.1. □

Theorem 2.5. *Let $h \geq 1$ be an integer. For any integer $m \geq 0$ the rigid analytic $\breve{K}$-variety $\mathcal{X}_m^{(h)}$ is connected and its ring $\mathcal{O}(\mathcal{X}_m^{(h)}) =: A_m^{(h)}$ of global sections is an integrally closed integral domain.*

Proof: The group $H_0^{(h)}/H_m^{(h)}$ acts transitively on the set of connected components of $\mathcal{X}_m^{(h)}$. Indeed, any connected component C of $\mathcal{X}_m^{(h)}$ is finite and flat over the connected space $\mathcal{X}_0^{(h)}$, hence maps surjectively onto $\mathcal{X}_0^{(h)}$ (cf. Proposition A.6). Thus, it suffices to see that $H_0^{(h)}$ acts transitively on the fibre in $\mathcal{X}_m^{(h)}$ of any point in $\mathcal{X}_0^{(h)}$. But here the transitivity follows from Theorem 2.2 and [3, V.2.2 Théorème 2].

Thus, choosing a connected component C of $\mathcal{X}_m^{(h)}$ and denoting by H' its stabilizer group in $H_0^{(h)}/H_m^{(h)}$, we need to show that $H' = H_0^{(h)}/H_m^{(h)}$. Since $H_0^{(h)}/H_m^{(h)}$ is a finite group, it suffices to show that it is the union of the conjugates of its subgroup H' (cf. [1, Exercice I.5.6, p. 130]).

Let $L|K$ be an extension of degree h and choose an embedding $L \hookrightarrow D = D_K^{(h)}$, inducing embeddings $L^* \hookrightarrow H^{(h)}$ and $\mathfrak{o}_L^\times \hookrightarrow H_0^{(h)}$. Denoting by $e = e_{L|K}$ the ramification index of the extension, we shall recall in Section 5 how to construct an $\mathfrak{o}_L^\times$-equivariant morphism $\mathcal{X}_{em,L}^{(1)} \to \mathcal{X}_m^{(h)} = \mathcal{X}_{m,K}^{(h)}$, where the index indicates which base field the objects refer to. The space $\mathcal{X}_{em,L}^{(1)} = \mathrm{Sp}(\breve{L}_{em})$ consists of just one point whose image in $\mathcal{X}_{m,K}^{(h)}$ we denote by y_L. By the above reasoning there is an element $\delta \in H_0^{(h)}$ such that $\delta \cdot y_L \in C$. It follows that for any extension L of K of degree h there is an embedding $L \hookrightarrow D$ such that the image of $\mathfrak{o}_L^\times$ in $H_0^{(h)}/H_m^{(h)}$ is contained in H'.

According to [15, §2], the action of $H_0^{(h)}$ on $\mathcal{X}_m^{(h)}$ extends semilinearly to the full group $H^{(h)}$. In particular, the action of $H_0^{(h)}$ on the set of connected components of $\mathcal{X}_m^{(h)}$ extends to an action of $H^{(h)}$. According to [15, §3], the morphism $\mathcal{X}_{em,L}^{(1)} \to \mathcal{X}_{m,K}^{(h)}$ is L^*-equivariant. Choosing L to be totally ramified over K, the point y_L is fixed by a uniformizer π_L of L, and the component C is fixed by the uniformizer $\Pi := \delta\pi_L\delta^{-1}$ of D. It follows that the subgroup H' of $H^{(h)}/H_m^{(h)}$ is normalized by the image of Π.

By abuse of notation, let $H' \subseteq H_0^{(h)}$ be any subgroup which is normalized by a suitable uniformizer Π of D and which contains a copy of $\mathfrak{o}_L^\times$ for any field extension L of K of degree h via a suitable embedding $L \hookrightarrow D$ (e.g. the inverse image of H' in $H_0^{(h)}$ under the projection $H_0^{(h)} \to H_0^{(h)}/H_m^{(h)}$). We show that $H_0^{(h)}$ is the union of the conjugates of H'. Indeed, let $\alpha \in H_0^{(h)}$ and let L' be a maximal commutative subfield of D containing $K[\alpha]$. According to [2, VIII.10.3 Corollaire à la Proposition 3], the field L' is of degree h over K. By the theorem of Skolem-Noether (cf. [2, VIII.10.1 Théorème 1]), there is an element $\delta \in H^{(h)}$ such that $L := \delta \cdot L' \cdot \delta^{-1}$ has the property that $\mathfrak{o}_L^\times \subset H'$. Writing $\delta = \Pi^r\delta_0$ with $\delta_0 \in H_0^{(h)}$ and a suitable integer r, we obtain $\alpha \in \delta_0^{-1}H'\delta_0$, because H' is normalized by Π.

The final assertion of the theorem is now a consequence of Theorem 2.2 and Lemma A.1. □

Remark 2.6. In the case where K is a local function field, Genestier constructed a $G_0^{(h)} \times H_0^{(h)}$-equivariant morphism $\mathcal{X}_m^{(h)} \to \mathrm{Sp}(\breve{K}_m)$ (cf. [21, Chapitre IV, §2]), the actions of $G_0^{(h)}$ and $H_0^{(h)}$ on $\breve{K}_m$ being as in Theorem 1.4. Letting N_m be the kernel of the map $H_0^{(h)}/H_m^{(h)} \to (\mathfrak{o}/\pi^m\mathfrak{o})^\times$ induced by the reduced norm $\mathrm{Nrd} : H_0^{(h)} \to \mathfrak{o}^\times$, the space $\mathcal{X}_m^{(h)}/N_m \simeq \mathcal{X}_0^{(h)} \times_{\breve{K}} \breve{K}_m$ can be thought of as being obtained by trivializing the determinant of the universal special formal $\mathfrak{o}_D$-

module on $\mathcal{X}_0^{(h)}$. The construction of an equivariant morphism $\mathcal{X}_m^{(h)} \to \mathrm{Sp}(\breve{K}_m)$ in characteristic zero (and for many other moduli spaces) is the subject of the forthcoming thesis [9] of Chen, as well as of the recent work [27] of Hedayatzadeh. For the Drinfeld tower, a global construction, relying on Carayol's strategy [8, Section 4.3], of computing the geometrically connected components of the spaces $\mathcal{X}_m^{(h)}$, was given in [5].

Proposition 2.7. *The field $\breve{K}_m$ is separably closed in $A_m^{(h)}$.*

Proof: Let E be the separable closure of $\breve{K}$ in $A_m^{(h)}$. Since $A_m^{(h)}$ is an integral domain (cf. Theorem 2.5) the field E is stable under the action of the group G' of elements in $G^{(h)}$ whose determinant is contained in $\mathfrak{o}^\times$ (cf. Section 4 for the extension of the action from $G_0^{(h)}$ to G'). Let K' denote the unramified extension of degree h of K, and denote by y the image of an $\mathfrak{o}_{K'}^\times$-equivariant morphism $\mathrm{Sp}(\breve{K}'_m) = \mathcal{X}^{(1)}_{m,K'} \to \mathcal{X}^{(h)}_{m,K}$ as in Section 5. Since $\breve{K}' = \breve{K}$, the Galois group of the extension $\kappa(y)|\breve{K}$ is a quotient of $\mathfrak{o}_{K'}^\times \subset G'$. Since E embeds $\mathfrak{o}_{K'}^\times$-equivariantly into $\kappa(y)$, it follows that $\mathrm{Gal}(E|\breve{K})$ is an abelian quotient of G'. Since the commutator subgroup of G' is $\mathrm{SL}_h(K)$ and since the determinant on $G^{(h)}$ restricts to the norm map $\mathrm{N}_{K'|K}$ on $(K')^*$, it follows that E is fixed by all elements $\alpha \in \mathfrak{o}_{K'}^\times$ such that $\mathrm{N}_{K'|K}(\alpha) = 1$. Consider the diagram

$$\begin{array}{ccc} \mathrm{Gal}(\breve{K}'_m|\breve{K}') & \xrightarrow{\text{res}} & \mathrm{Gal}(\breve{K}_m|\breve{K}) \\ \downarrow \simeq & & \simeq \downarrow \\ (\mathfrak{o}_{K'}/\pi^m\mathfrak{o}_{K'})^\times & \xrightarrow{\mathrm{N}_{K'|K}} & (\mathfrak{o}/\pi^m\mathfrak{o})^\times, \end{array} \tag{7}$$

which is commutative according to the base change property of local class field theory. In fact, for abelian extensions generated by torsion points of one dimensional Lubin-Tate modules of height one, it can be proved directly (cf. [44, Theorem 5.9]). It follows that $E \subseteq \breve{K}_m$. $\square$

We are now ready to prove an analog of Theorem 1.4.

Theorem 2.8. *Let $h \geq 1$ and $m \geq 0$ be integers.*

(i) *We have $(A_m^{(h)})^{H_0^{(h)}} = A_0^{(h)}$.*

(ii) *We have $(A_m^{(h)})^{G_m^{(h)}} = \breve{K}_m$, and the resulting actions of $G_0^{(h)}$ and $H_0^{(h)}$ on $\breve{K}_m$ are as in Theorem 1.4.*

(iii) *If M is a finitely generated projective $A_m^{(h)}$-module with a semilinear action of $H_0^{(h)}/H_m^{(h)}$, then the natural map $A_m^{(h)} \otimes_{A_0^{(h)}} M^{H_0^{(h)}} \to M$ is an isomorphism.*

Proof: Assertion (i) is a direct consequence of Theorem 2.2. As in Theorem 1.4, assertion (ii) follows from Proposition 2.7 together with the following lemma. Finally, assertion (iii) follows from Corollary A.3 and Theorem A.4. $\square$

Lemma 2.9. *We have $(A_0^{(h)})^G = \breve{K}$ for any open subgroup G of $G_0^{(h)}$.*

Proof: Let $z = [z_0 : \ldots : z_{h-1}] \in \mathcal{X}_0^{(h)} \subset \mathbb{P}^{h-1}_{\breve{K}}$ be a $\breve{K}$-rational point (for example the image in $\mathcal{X}_0^{(h)}$ of the point y which appears in the proof of Proposition 2.7). Multiplying z by suitable elementary matrices with entries 0 or 1, we obtain a $\breve{K}$-rational point all of whose coordinates are different from zero. We again denote it by z.

Let $f \in A_0^{(h)}$ be G-invariant. Replacing f by $f - f(z)$, we may assume $f(z) = 0$. There is an integer $m \geq 0$ such that also $f(z') = 0$ for any point z' of the form $z' = [\alpha_0 z_0 : \ldots : \alpha_{h-1} z_{h-1}]$ with $\alpha_i \in 1 + \pi^m \mathfrak{o}$. Since $z_i \neq 0$ for each index i, the subset $(1 + \pi^m \mathfrak{o}) z_i$ of $\breve{K}$ has z_i as a limit point. Therefore, an elementary induction argument on h shows that we must have $f = 0$. $\square$

The following corollary can be proved like Corollary 1.6.

Corollary 2.10. *Let $m \geq 0$ be an integer. We have $(A_m^{(h)})^G = (\breve{K}_m)^{\det(G)}$ for any open subgroup G of $G_0^{(h)}$.* $\square$

3 Admissible bundles on the deformation spaces

Let $h \geq 1$ and $m \geq 0$ be integers, and set $\mathcal{Z}_m^{(h)} := \mathcal{X}_m^{(h)} \times_{\breve{K}_m} \mathcal{Y}_m^{(h)}$. The morphisms (4) and (6) induce morphisms

$$\mathcal{Z}_{m'}^{(h)} \longrightarrow \mathcal{Z}_m^{(h)} \tag{8}$$

for any integer m' with $m' \geq m$. Set $C_m^{(h)} := \mathcal{O}(\mathcal{Z}_m^{(h)})$. Combining Theorem 1.2, Theorem 2.2 and Theorem 2.5, we obtain the following results.

Theorem 3.1. *Let $h \geq 1$ be an integer. For any integer $m \geq 0$ the rigid analytic $\breve{K}$-variety $\mathcal{Z}_m^{(h)}$ is smooth connected and quasi-Stein. In particular, the ring $C_m^{(h)}$ is an integrally closed integral domain. If m' is an integer such that $m' \geq m$ then the morphism* (8) *is finite étale and Galois.*

Proof: According to [41, Theorem 4.4], the space $\mathcal{Y}_m^{(h)}$ is geometrically connected over $\mathrm{Sp}(\breve{K}_m)$. The connectedness of $\mathcal{Z}_m^{(h)}$ is therefore a consequence of Theorem 2.5 and [16, Théorème 8.4].

The smoothness of $\mathcal{Z}_m^{(h)}$ follows from the corresponding properties of $\mathcal{X}_m^{(h)}$ and $\mathcal{Y}_m^{(h)}$ (cf. Theorem 1.2 and Theorem 2.2), as well as from [11, Theorem 4.2.7]. The last reference also implies the covering morphisms (8) to be étale. They are finite by the remark following [6, 9.4.4 Corollary 2]. Finally, the property of being Galois is also a formal consequence of the corresponding fact for the morphisms (4) and (6). Since we will not make use of this result, we leave the details to the reader.

Finally, the assertions concerning the ring $C_m^{(h)}$ follow from Lemma A.1.$\square$

For any two integers $h \geq 1$ and $m \geq 0$ the rigid analytic $\breve{K}_m$-varieties $\mathcal{X}_m^{(h)}$ and $\mathcal{Y}_m^{(h)}$ admit admissible coverings by increasing sequences of affinoid subdomains such that the inclusion maps are relatively compact over $\breve{K}_m$ in the sense of [6, Section 9.6.2], and such that the restriction maps on the corresponding affinoid algebras have dense image. It follows from [17, Proposition 2.1.16] and [36, Propositions 16.5 and 20.7], that $A_m^{(h)}$ and $B_m^{(h)}$ are nuclear $\breve{K}_m$-Fréchet spaces in the sense of [36, §19].

If V and W are two locally convex vector spaces over a complete nonarchimedean valuation field F then we denote by $V\hat{\otimes}_F W$ the complete projective tensor product of V and W over F (cf. [36, §17]).

It follows from the above results as well as from [6, 9.6.2 Lemma 1], [17, Proposition 1.1.29], and [36, Corollary 20.14], that $C_m^{(h)}$ is a nuclear $\breve{K}_m$-Fréchet space and that there is a natural topological isomorphism

$$C_m^{(h)} \simeq A_m^{(h)} \hat{\otimes}_{\breve{K}_m} B_m^{(h)}.$$

Note that by a cofinality argument the topologies of $A_m^{(h)}$, $B_m^{(h)}$ and $C_m^{(h)}$ do not depend on the choice of the admissible affinoid coverings chosen above. It follows that the groups $G_0^{(h)}$ and $H_0^{(h)}$ act on $A_m^{(h)}$ and $B_m^{(h)}$ by continuous $\breve{K}$-linear automorphisms. According to Theorems 1.4 and 2.8 the actions of $G_0^{(h)}$ and $H_0^{(h)}$ on the common subalgebra $\breve{K}_m$ of $A_m^{(h)}$ and $B_m^{(h)}$ agree. By continuity, we obtain commuting diagonal left actions of $G_0^{(h)}$ and $H_0^{(h)}$ on $C_m^{(h)}$.

Theorem 3.2. *For any two integers $h \geq 1$ and $m \geq 0$ there are isomorphisms $(C_m^{(h)})^{H_0^{(h)}} \simeq A_0^{(h)}$ and $(C_m^{(h)})^{G_0^{(h)}} \simeq B_0^{(h)}$ which are $G_0^{(h)}$-equivariant and $H_0^{(h)}$-equivariant, respectively.*

Proof: Given the results of Theorem 1.4 and Theorem 2.8, the two assertions follow from the following general fact by first considering the invariants under the open subgroups $H_m^{(h)}$ and $G_m^{(h)}$, respectively. □

Lemma 3.3. *Let F be a field which is spherically complete with respect to a nonarchimedean valuation, and let V and W be F-Fréchet spaces. Let Γ be a group acting on V by continuous F-linear automorphisms and endow W with the trivial Γ-action. If one of V or W is nuclear in the sense of* [36, §19]*, then*

$$(V\hat{\otimes}_F W)^\Gamma \simeq V^\Gamma \hat{\otimes}_F W.$$

Proof: Denote by W' the space of continuous F-linear functionals on W. Given a nonzero element $\lambda \in W'$ endow the quotient $W_\lambda := W/\ker(\lambda)$ with the quotient topology which makes it a one dimensional F-vector space with its natural topology. Due to the Hahn-Banach theorem, the natural map

$$W \to \prod_{\lambda\in W'} W_\lambda$$

is continuous and injective (cf. [36, Corollary 9.3]). Endow V/V^Γ with the quotient topology and consider the commutative diagram

$$\begin{array}{ccccccccc}
0 & \longrightarrow & V^\Gamma\hat{\otimes}_F W & \longrightarrow & V\hat{\otimes}_F W & \longrightarrow & (V/V^\Gamma)\hat{\otimes}_F W & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \prod_{\lambda\in W'} V^\Gamma \otimes_F W_\lambda & \longrightarrow & \prod_{\lambda\in W'} V \otimes_F W_\lambda & \longrightarrow & \prod_{\lambda\in W'} (V/V^\Gamma) \otimes_F W_\lambda & \longrightarrow & 0.
\end{array}$$

We shall need several properties of the complete projective tensor product over F which can be proved as in the archimedean context. Notably, the complete projective tensor product commutes with arbitrary direct products (cf. [24, I.1.3 Proposition 6]). It coincides with the usual tensor product if both spaces are Hausdorff and if one of the factors is finite dimensional over F, endowed with its natural topology. It follows from the nuclearity and metrizability assumptions, as well as from [24, I.1.2 Proposition 3 and II.3.1 Corollaire à la Proposition 10], that the two rows in the above diagram are exact. The vertical arrows are injective according to [24, I.1.2 Proposition 3]. Since the middle arrow is Γ-equivariant and since $(V \otimes_F W_\lambda)^\Gamma \simeq V^\Gamma \otimes_F W_\lambda$ for all $\lambda \in W'$, the result follows. $\square$

Let $\mathcal{B}_{\mathcal{X}_0^{(h)}}(G_0^{(h)})$ denote the category of $G_0^{(h)}$-equivariant vector bundles of finite rank on $\mathcal{X}_0^{(h)}$. Recall from Corollary A.3 that the global section functor is an equivalence between $\mathcal{B}_{\mathcal{X}_0^{(h)}}(G_0^{(h)})$ and the category of finitely generated projective $A_0^{(h)}$-modules with a semilinear action of $G_0^{(h)}$. Given such a module M, we denote by $\tilde{M}$ the associated equivariant vector bundle. If $m \geq 0$ is an integer, we let the group $G_0^{(h)}$ act diagonally on $C_m^{(h)} \otimes_{A_0^{(h)}} M$.

Similarly, let $\mathcal{B}_{\mathcal{Y}_0^{(h)}}(H_0^{(h)})$ denote the category of $H_0^{(h)}$-equivariant vector bundles of finite rank on $\mathcal{Y}_0^{(h)}$. Results and conventions analogous to those for $\mathcal{B}_{\mathcal{X}_0^{(h)}}(G_0^{(h)})$ apply.

Definition 3.4. (i) A $G_0^{(h)}$-equivariant vector bundle $\mathcal{M} = \tilde{M}$ of finite rank on $\mathcal{X}_0^{(h)}$ is called *Lubin-Tate* if there is an integer $m \geq 0$ such that the natural map

$$C_m^{(h)} \otimes_{B_0^{(h)}} (C_m^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}} \longrightarrow C_m^{(h)} \otimes_{A_0^{(h)}} M \tag{9}$$

is an isomorphism.

(ii) An $H_0^{(h)}$-equivariant vector bundle $\mathcal{N} = \tilde{N}$ of finite rank on $\mathcal{Y}_0^{(h)}$ is called *Drinfeld* if there is an integer $m \geq 0$ such that the natural map

$$C_m^{(h)} \otimes_{A_0^{(h)}} (C_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_0^{(h)}} \longrightarrow C_m^{(h)} \otimes_{B_0^{(h)}} N \tag{10}$$

is an isomorphism.

Assume that $\mathcal{M} = \tilde{M}$ is a Lubin-Tate bundle on $\mathcal{X}_0^{(h)}$. Choose an integer $m \geq 0$ as in Definition 3.4 and let $x \in \mathcal{X}_m^{(h)}$ be any point. Then x corresponds to a closed maximal ideal $\mathfrak{m}$ of $A_m^{(h)}$. One can show that $\mathfrak{m}$ is of finite codimension over $\breve{K}_m$, so that the sequence

$$0 \longrightarrow \mathfrak{m}\hat{\otimes}_{\breve{K}_m} B_m^{(h)} \longrightarrow C_m^{(h)} \longrightarrow \kappa(x) \otimes_{\breve{K}_m} B_m^{(h)} \longrightarrow 0$$

is exact. We obtain from (9) that the natural map

$$(\kappa(x) \otimes_{\breve{K}_m} B_m^{(h)}) \otimes_{B_0^{(h)}} (C_m^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}} \longrightarrow (\kappa(x) \otimes_{\breve{K}_m} B_m^{(h)}) \otimes_{A_0^{(h)}} M$$

is an isomorphism. Since the right-hand side is a finitely generated projective module over $\kappa(x) \otimes_{\breve{K}_m} B_m^{(h)}$ and since the latter is faithfully flat over $B_0^{(h)}$ (cf. Theorem 1.2 and Proposition A.5), it follows that $(C_m^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}}$ is a finitely generated projective $B_0^{(h)}$-module of the same rank as M. Moreover, via the action of $H_0^{(h)}$ on $C_m^{(h)}$, which commutes with that of $G_0^{(h)}$ and is trivial on $A_0^{(h)}$, we obtain an $H_0^{(h)}$-equivariant vector bundle

$$\mathbb{D}_{\mathrm{LT}}(\mathcal{M}) := [(C_m^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}}]^{\sim} \tag{11}$$

of finite rank on $\mathcal{Y}_0^{(h)}$. Similarly, if $\mathcal{N} = \tilde{N}$ is a Drinfeld bundle on $\mathcal{Y}_0^{(h)}$ and if the integer $m \geq 0$ is as in Definition 3.4 then

$$\mathbb{D}_{\mathrm{Dr}}(\mathcal{N}) := [(C_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_0^{(h)}}]^{\sim} \tag{12}$$

is a $G_0^{(h)}$-equivariant vector bundle on $\mathcal{X}_0^{(h)}$ of the same rank as $\mathcal{N}$.

Lemma 3.5. (i) *If $\mathcal{M} = \tilde{M}$ is a $G_0^{(h)}$-equivariant vector bundle of finite rank on $\mathcal{X}_0^{(h)}$ such that the map (9) is bijective for some integer $m \geq 0$ then it is also bijective for any integer $m' \geq m$, and the natural homomorphism $C_m^{(h)} \otimes_{A_0^{(h)}} M \to C_{m'}^{(h)} \otimes_{A_0^{(h)}} M$ induces an isomorphism on $G_0^{(h)}$-invariants.*

(ii) *If $\mathcal{N} = \tilde{N}$ is an $H_0^{(h)}$-equivariant vector bundle of finite rank on $\mathcal{Y}_0^{(h)}$ such that the map (10) is bijective for some integer $m \geq 0$ then it is also bijective for any integer $m' \geq m$, and the natural homomorphism $C_m^{(h)} \otimes_{B_0^{(h)}} N \to C_{m'}^{(h)} \otimes_{B_0^{(h)}} N$ induces an isomorphism on $H_0^{(h)}$-invariants.*

Proof: As for (ii), note that the isomorphism (10) is $H_0^{(h)}$-equivariant if we let $H_0^{(h)}$ act on $C_m^{(h)}$ on the left and diagonally on the right-hand side. Tensoring with $C_{m'}^{(h)}$ over $C_m^{(h)}$ and passing to $H_0^{(h)}$-invariants, we obtain the isomorphism

$$(C_{m'}^{(h)} \otimes_{B_0^{(h)}} N)^{H_0^{(h)}} \simeq (C_{m'}^{(h)} \otimes_{A_0^{(h)}} (C_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_0^{(h)}})^{H_0^{(h)}}.$$

Since $(C_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_0^{(h)}}$ is a projective $A_0^{(h)}$-module, it follows from Theorem 3.2 that the right-hand side is naturally isomorphic to

$$(C_{m'}^{(h)})^{H_0^{(h)}} \otimes_{A_0^{(h)}} (C_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_0^{(h)}} \simeq (C_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_0^{(h)}},$$

proving the second claim. The first claim is obtained by tensoring with $C_{m'}^{(h)}$ over $A_0^{(h)}$ and once again using (10) for the integer m. Assertion (i) can be proved analogously. □

Corollary 3.6. *If $\mathcal{M}$ and $\mathcal{N}$ are Lubin-Tate and Drinfeld bundles on $\mathcal{X}_0^{(h)}$ and $\mathcal{Y}_0^{(h)}$, respectively, then the vector bundles $\mathbb{D}_{\mathrm{LT}}(\mathcal{M})$ and $\mathbb{D}_{\mathrm{Dr}}(\mathcal{N})$ are independent of the integers $m \geq 0$ appearing in Definition* 3.4. □

Denote by $\mathcal{B}^{\mathrm{LT}}_{\mathcal{X}_0^{(h)}}(G_0^{(h)})$ and $\mathcal{B}^{\mathrm{Dr}}_{\mathcal{Y}_0^{(h)}}(H_0^{(h)})$ the full subcategories of $\mathcal{B}_{\mathcal{X}_0^{(h)}}(G_0^{(h)})$ and $\mathcal{B}_{\mathcal{Y}_0^{(h)}}(H_0^{(h)})$ consisting of all Lubin-Tate and Drinfeld bundles, respectively.

Theorem 3.7. *If $\mathcal{M}$ is a Lubin-Tate bundle on $\mathcal{X}_0^{(h)}$ then $\mathbb{D}_{\mathrm{LT}}(\mathcal{M})$ is a Drinfeld bundle on $\mathcal{Y}_0^{(h)}$. If $\mathcal{N}$ is a Drinfeld bundle on $\mathcal{Y}_0^{(h)}$ then $\mathbb{D}_{\mathrm{Dr}}(\mathcal{N})$ is a Lubin-Tate bundle on $\mathcal{X}_0^{(h)}$. The assignments*

$$\mathbb{D}_{\mathrm{LT}} := (\mathcal{M} \mapsto \mathbb{D}_{\mathrm{LT}}(\mathcal{M})) \quad \textit{and} \quad \mathbb{D}_{\mathrm{Dr}} := (\mathcal{N} \mapsto \mathbb{D}_{\mathrm{Dr}}(\mathcal{N}))$$

are mutually quasi-inverse equivalences of categories between $\mathcal{B}^{\mathrm{LT}}_{\mathcal{X}_0^{(h)}}(G_0^{(h)})$ and $\mathcal{B}^{\mathrm{Dr}}_{\mathcal{Y}_0^{(h)}}(H_0^{(h)})$.

Proof: Let $\mathcal{M} = \tilde{M}$ be a Lubin-Tate bundle on $\mathcal{X}_0^{(h)}$. The isomorphism (9) of $C_m^{(h)}$-modules is $H_0^{(h)}$-equivariant if we let $H_0^{(h)}$ act diagonally on the left-hand side and via its action on $C_m^{(h)}$ on the right-hand side. Since M is a projective $A_0^{(h)}$-module we have

$$(C_m^{(h)} \otimes_{A_0^{(h)}} M)^{H_0^{(h)}} \simeq (C_m^{(h)})^{H_0^{(h)}} \otimes_{A_0^{(h)}} M \simeq M$$

by Theorem 3.2. It follows that $\mathbb{D}_{\mathrm{LT}}(\mathcal{M})$ is Drinfeld and that $\mathbb{D}_{\mathrm{Dr}}(\mathbb{D}_{\mathrm{LT}}(\mathcal{M})) \simeq \mathcal{M}$, naturally in $\mathcal{M}$.

Similarly, one can show that $\mathbb{D}_{\mathrm{Dr}}(\mathcal{N})$ is Lubin-Tate if $\mathcal{N}$ is Drinfeld and that in this case $\mathbb{D}_{\mathrm{LT}}(\mathbb{D}_{\mathrm{Dr}}(\mathcal{N})) \simeq \mathcal{N}$, naturally in $\mathcal{N}$. Since the assignments $\mathbb{D}_{\mathrm{LT}}$ and $\mathbb{D}_{\mathrm{Dr}}$ are obviously functorial, the theorem is proved. $\square$

A large class of Lubin-Tate and Drinfeld bundles is provided by the following construction. Denote by $\mathrm{Rep}^\infty_{\breve{K}}(G_0^{(h)})$ and $\mathrm{Rep}^\infty_{\breve{K}}(H_0^{(h)})$ the categories of smooth representations of $G_0^{(h)}$ and $H_0^{(h)}$ on finite dimensional $\breve{K}$-vector spaces, respectively. If V is an object of $\mathrm{Rep}^\infty_{\breve{K}}(G_0^{(h)})$ then, via the diagonal $G_0^{(h)}$-action, $\mathcal{M}(V) := (A_0^{(h)} \otimes_{\breve{K}} V)^\sim$ is a $G_0^{(h)}$-equivariant vector bundle of finite rank on $\mathcal{X}_0^{(h)}$. Similarly, $\mathcal{N}(W) := (B_0^{(h)} \otimes_{\breve{K}} W)^\sim$ is an $H_0^{(h)}$-equivariant vector bundle of finite rank on $\mathcal{Y}_0^{(h)}$ for any object W of $\mathrm{Rep}^\infty_{\breve{K}}(H_0^{(h)})$.

Theorem 3.8. *If V and W are objects of $\mathrm{Rep}^\infty_{\breve{K}}(G_0^{(h)})$ and $\mathrm{Rep}^\infty_{\breve{K}}(H_0^{(h)})$, respectively, then the equivariant vector bundles $\mathcal{M}(V)$ and $\mathcal{N}(W)$ are Lubin-Tate and Drinfeld, respectively.*

Proof: Since $\dim_{\breve{K}}(V) < \infty$ there is an integer $m \geq 0$ such that the action of $G_0^{(h)}$ on V factors through $G_0^{(h)}/G_m^{(h)}$. By Theorem 1.4 the natural map

$$B_m^{(h)} \otimes_{B_0^{(h)}} (B_m^{(h)} \otimes_{\breve{K}} V)^{G_0^{(h)}} \longrightarrow B_m^{(h)} \otimes_{\breve{K}} V \tag{13}$$

is bijective. Note that by Lemma 3.3 and Theorem 2.8

$$(C_m^{(h)} \otimes_{\breve{K}} V)^{G_0^{(h)}} \simeq ((C_m^{(h)})^{G_m^{(h)}} \otimes_{\breve{K}} V)^{G_0^{(h)}} \simeq (B_m^{(h)} \otimes_{\breve{K}} V)^{G_0^{(h)}}.$$

Therefore, tensoring (13) with $C_m^{(h)}$ over $B_m^{(h)}$, we obtain that $\mathcal{M}(V)$ is Lubin-Tate. That the $H_0^{(h)}$-equivariant vector bundle $\mathcal{N}(W)$ is Drinfeld follows by a similar reasoning. $\square$

Remark 3.9. If V is a finite dimensional smooth representation of $G_0^{(h)}$ over $\breve{K}$ then the $H_0^{(h)}$-equivariant vector bundle $\mathbb{D}_{\mathrm{LT}}(\mathcal{M}(V))$ on $\mathcal{Y}_0^{(h)}$ is typically *not* of the form $\mathcal{N}(W)$ for any object W of $\mathrm{Rep}_{\breve{K}}^{\infty}(H_0^{(h)})$. If for example $m \geq 0$ is an integer and if $V := \breve{K}[G_0^{(h)}/G_m^{(h)}]$ with $G_0^{(h)}$ acting through the left regular representation, then one can check that $\mathbb{D}_{\mathrm{LT}}(\mathcal{M}(V)) \simeq \mathcal{O}_{\mathcal{Y}_m^{(h)}}$. Likewise, $\mathbb{D}_{\mathrm{Dr}}(\mathcal{N}(\breve{K}[H_0^{(h)}/H_m^{(h)}])) \simeq \mathcal{O}_{\mathcal{X}_m^{(h)}}$ for any integer $m \geq 0$.

We are now going to study the formal properties of the categories of Lubin-Tate and Drinfeld bundles.

Let B be a ring carrying the action of a group Γ. Assume $E := B^{\Gamma}$ to be a field and let $F \subseteq E$ be a subfield. Recall from [20, Definition 2.8], that B is called (F, Γ)-*regular* if B is an integral domain such that $\mathrm{Quot}(B)^{\Gamma} = B^{\Gamma}$ and such that any element $f \in B$ spanning a one dimensional Γ-stable F-subspace of B is a unit.

Lemma 3.10. *For any integer $m \geq 0$ the rings $A_m^{(h)}$ and $B_m^{(h)}$ are $(\breve{K}_m, G_m^{(h)})$-regular and $(\breve{K}_m, H_m^{(h)})$-regular, respectively.*

Proof: Note first that $B_m^{(h)}$ is an integral domain and that $(B_m^{(h)})^{H_m^{(h)}} = \breve{K}_m$ by Theorems 1.2 and 1.4.

We will first show that $\mathrm{Quot}(B_0^{(h)})^{H_m^{(h)}} = \breve{K}$ for any integer $m \geq 0$. Since $B_0^{(h)}$ is integrally closed in its field of fractions, Lemma 1.5 shows that it suffices to treat the case $m = 0$. Let $f_1, f_2 \in B_0^{(h)}$ with f_2 nonzero, such that $F := f_1/f_2 \in \mathrm{Quot}(B_0^{(h)})$ is $H_0^{(h)}$-invariant. According to the proof of [23, Proposition 14.18], there is a $\breve{K}$-rational point $y \in \mathcal{Y}_0^{(h)}$ whose $H_0^{(h)}$-orbit is Zariski dense. Thus, $f_2(y) \neq 0$. Set $\alpha := f_1(y)/f_2(y) \in \breve{K}$ and consider $F' := f_1 - \alpha f_2 \in B_0^{(h)}$. It follows from the $H_0^{(h)}$-invariance of F that $F'(y') = 0$ for all $y' \in H_0^{(h)} y$. Thus, $F' = 0$ and $F = \alpha \in \breve{K}$.

In the general case, $\mathrm{Quot}(B_m^{(h)})^{H_m^{(h)}}$ is integral over $\mathrm{Quot}(B_0^{(h)})^{H_m^{(h)}} = \breve{K} \subset B_m^{(h)}$. Since $B_m^{(h)}$ is integrally closed in its field of fractions (cf. Theorem 1.2) we have $\mathrm{Quot}(B_m^{(h)})^{H_m^{(h)}} = (B_m^{(h)})^{H_m^{(h)}}$.

Now assume $f \in B_0^{(h)}$ to span an $H_m^{(h)}$-stable one dimensional $\breve{K}$-subspace of $B_0^{(h)}$ for some integer $m \geq 0$. We show that f is a unit. Let $\{h_1, \ldots, h_n\}$ be a set of representatives of $H_0^{(h)}/H_m^{(h)}$ in $H_0^{(h)}$ and set $\tilde{f} := \prod_i h_i \cdot f$. The element $\tilde{f}$ of $B_0^{(h)}$ is nonzero and spans an $H_0^{(h)}$-stable subspace. Therefore, the action of $H_0^{(h)}$ on $\tilde{f}$ is given by a character $\chi : H_0^{(h)} \to \breve{K}^*$. According to Lemma 1.5 it will suffice to show that χ is trivial.

According to the proof of [23, Proposition 14.18], the stabilizer group of the above point $y \in \mathcal{Y}_0^{(h)}$ is the group of units $\mathfrak{o}_{K'}^{\times}$ of the valuation ring of the unramified extension K' of degree h of K via some embedding $\mathfrak{o}_{K'}^{\times} \hookrightarrow H_0^{(h)}$. It follows from our assumptions that the image of $\tilde{f}$ in $\kappa(y) = \breve{K}$ is nonzero. Since the induced action of $\mathfrak{o}_{K'}^\times$ on $\kappa(y)$ is trivial, it follows that $\chi|\mathfrak{o}_{K'}^\times = 1$.

Further, the restriction of the reduced norm map $\mathrm{Nrd} : H_0^{(h)} \to \mathfrak{o}^\times$ to $\mathfrak{o}_{K'}^\times$ is surjective. Together with [34, Corollary 4.1.2], this implies that any element of $H_0^{(h)}$ is a product of an element in $\mathfrak{o}_{K'}^\times$ and a commutator in $H_0^{(h)}$. Thus, $\chi = 1$.

In the general case let $f \in B_m^{(h)}$ span a one dimensional $H_m^{(h)}$-stable $\breve{K}_m$-subspace and consider the norm $\tilde{f} := \mathrm{N}_{B_m^{(h)}|B_0^{(h)}}(f) = \prod_{g\in G_0^{(h)}/G_m^{(h)}} g\cdot f$ of f in $B_0^{(h)}$. Since the actions of $G_0^{(h)}$ and $H_0^{(h)}$ on $B_m^{(h)}$ commute and since the restriction of $\mathrm{N}_{B_m^{(h)}|B_0^{(h)}}$ to $\breve{K}_m$ is a power of $\mathrm{N}_{\breve{K}_m|\breve{K}}$ (cf. Theorem 1.4) it follows that $\tilde{f}$ spans a one dimensional $H_m^{(h)}$-stable $\breve{K}$-subspace in $B_0^{(h)}$. By the above reasoning we have $\tilde{f} \in (B_0^{(h)})^\times$ and thus $f(\prod_{g\neq 1} g\cdot f)\tilde{f}^{-1} = 1$.

The ring $A_m^{(h)}$ can be treated similarly. □

Lemma 3.11. (i) *For any $G_0^{(h)}$-equivariant vector bundle $\mathcal{M} = \tilde{M}$ of finite rank on $\mathcal{X}_0^{(h)}$ and for any integer $m \geq 0$ the natural map*

$$A_m^{(h)} \otimes_{\breve{K}_m} (A_m^{(h)} \otimes_{A_0^{(h)}} M)^{G_m^{(h)}} \longrightarrow A_m^{(h)} \otimes_{A_0^{(h)}} M \tag{14}$$

is injective. The vector bundle $\mathcal{M}$ is Lubin-Tate if and only if there is an integer $m \geq 0$ such that the map (14) is bijective.

(ii) *For any $H_0^{(h)}$-equivariant vector bundle $\mathcal{N} = \tilde{N}$ of finite rank on $\mathcal{Y}_0^{(h)}$ and for any integer $m \geq 0$ the natural map*

$$B_m^{(h)} \otimes_{\breve{K}_m} (B_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_m^{(h)}} \longrightarrow B_m^{(h)} \otimes_{B_0^{(h)}} N \tag{15}$$

is injective. The vector bundle $\mathcal{N}$ is Drinfeld if and only if there is an integer $m \geq 0$ such that the map (15) is bijective.

Proof: Using Lemma 3.10 the injectivity of the maps (14) and (15) can be proved as in [20, Theorem 2.13].

If (15) is a bijection then $W := (B_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_m^{(h)}}$ is a finite dimensional $\breve{K}_m$-vector space carrying a semilinear action of $H_0^{(h)}/H_m^{(h)}$. By Theorem 2.8 the natural map

$$A_m^{(h)} \otimes_{A_0^{(h)}} (A_m^{(h)} \otimes_{\breve{K}_m} W)^{H_0^{(h)}} \longrightarrow A_m^{(h)} \otimes_{\breve{K}_m} W \tag{16}$$

is an isomorphism. Since it is $A_m^{(h)}$-linear it is even a topological isomorphism with respect to certain natural Fréchet topologies on both sides (cf. the remarks preceding [40, Proposition 3.7]). Taking the complete tensor product with $B_m^{(h)}$ over $\breve{K}_m$, we obtain an isomorphism

$$C_m^{(h)} \otimes_{A_0^{(h)}} (A_m^{(h)} \otimes_{\breve{K}_m} W)^{H_0^{(h)}} \longrightarrow A_m^{(h)} \hat{\otimes}_{\breve{K}_m} B_m^{(h)} \otimes_{\breve{K}_m} W.$$

Likewise, the map (15) is a topological isomorphism so that the right-hand side can be identified with $C_m^{(h)} \otimes_{B_0^{(h)}} N$. By Lemma 3.3 we have

$$(A_m^{(h)} \otimes_{\breve{K}_m} W)^{H_0^{(h)}} \simeq (C_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_0^{(h)}}, \tag{17}$$

and the above isomorphism turns out to be the natural homomorphism

$$C_m^{(h)} \otimes_{A_0^{(h)}} (C_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_0^{(h)}} \longrightarrow C_m^{(h)} \otimes_{B_0^{(h)}} N.$$

Therefore, $\mathcal{N}$ is Drinfeld.

Conversely, if (10) is an isomorphism for some integer $m \geq 0$ then, passing to $H_m^{(h)}$-invariants on both sides and using Theorem 1.4 and Lemma 3.3, we obtain

$$A_m^{(h)} \otimes_{\breve{K}_m} (B_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_m^{(h)}} \simeq (C_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_m^{(h)}} \simeq A_m^{(h)} \otimes_{A_0^{(h)}} (C_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_0^{(h)}}.$$

Here we used that $(B_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_m^{(h)}}$ is a finite dimensional $\breve{K}_m$-vector space because of the injectivity of (15). Tensoring with $C_m^{(h)}$ over $A_m^{(h)}$ and using (10) again, we obtain that the natural map

$$C_m^{(h)} \otimes_{\breve{K}_m} (B_m^{(h)} \otimes_{B_0^{(h)}} N)^{H_m^{(h)}} \longrightarrow C_m^{(h)} \otimes_{B_0^{(h)}} N$$

is bijective. As seen before, the ring $C_m^{(h)}$ has a quotient which is faithfully flat over $B_m^{(h)}$. Thus, we can deduce that (15) is bijective for the integer m. The analogous assertion in (i) can be proved similarly. □

As a consequence of the two preceding lemmas we obtain the following result.

Theorem 3.12. *The categories of Lubin-Tate and Drinfeld bundles are strictly full subcategories of* $\mathcal{B}_{\mathcal{X}_0^{(h)}}(G_0^{(h)})$ *and* $\mathcal{B}_{\mathcal{Y}_0^{(h)}}(H_0^{(h)})$, *respectively, which are closed under direct sums, tensor products and duals. The equivalences* $\mathbb{D}_{\mathrm{LT}}$ *and* $\mathbb{D}_{\mathrm{Dr}}$ *commute with these structures. Let*

$$0 \longrightarrow \mathcal{M}_1 \longrightarrow \mathcal{M}_2 \longrightarrow \mathcal{M}_3 \longrightarrow 0$$

be a sequence of homomorphisms of $G_0^{(h)}$*-equivariant vector bundles of finite rank on* $\mathcal{X}_0^{(h)}$ *and assume* $\mathcal{M}_2$ *to be Lubin-Tate. If the sequence is exact on the left (resp. on the right) then* $\mathcal{M}_1$ *(resp.* $\mathcal{M}_3$*) is Lubin-Tate, as well, and the induced sequence*

$$0 \longrightarrow \mathbb{D}_{\mathrm{LT}}(\mathcal{M}_1) \longrightarrow \mathbb{D}_{\mathrm{LT}}(\mathcal{M}_2) \longrightarrow \mathbb{D}_{\mathrm{LT}}(\mathcal{M}_3) \longrightarrow 0 \tag{18}$$

is exact on the left (resp. on the right). Analogous results hold for sequences in $\mathcal{B}_{\mathcal{Y}_0^{(h)}}(H_0^{(h)})$.

Proof: The properties of being strictly full and of admitting direct sums are clear. It is also clear that the functors $\mathbb{D}_{\mathrm{LT}}$ and $\mathbb{D}_{\mathrm{Dr}}$ commute with direct sums. Given an exact sequence of $G_0^{(h)}$-equivariant vector bundles of finite rank on $\mathcal{X}_0^{(h)}$ as above, write $\mathcal{M}_i = \tilde{M}_i$ for $i = 1, 2, 3$, let $m \geq 0$ be an integer, set $W_i := (A_m^{(h)} \otimes_{A_0^{(h)}} M_i)^{G_m^{(h)}}$, and consider the commutative diagram

$$\begin{tikzcd}
0 \arrow[r] & A_m^{(h)} \otimes_{\breve{K}_m} W_1 \arrow[r] \arrow[d, hook] & A_m^{(h)} \otimes_{\breve{K}_m} W_2 \arrow[r] \arrow[d, hook] & A_m^{(h)} \otimes_{\breve{K}_m} W_3 \arrow[d, hook] & \\
0 \arrow[r] & A_m^{(h)} \otimes_{A_0^{(h)}} M_1 \arrow[r] & A_m^{(h)} \otimes_{A_0^{(h)}} M_2 \arrow[r] & A_m^{(h)} \otimes_{A_0^{(h)}} M_3 \arrow[r] & 0.
\end{tikzcd}$$

The vertical maps are injective according to Lemma 3.11. Using the flatness of the ring homomorphism $A_0^{(h)} \to A_m^{(h)}$ (cf. Proposition A.5), it is straightforward to check that together with the vertical arrow in the middle also the one on the left (resp. on the right) is bijective once the initial sequence is exact on the left (resp. on the right). In this situation it follows that also the sequence

$$0 \longrightarrow W_1 \longrightarrow W_2 \longrightarrow W_3 \longrightarrow 0$$

is exact on the left (resp. on the right). Tensorizing with $B_m^{(h)}$ over $\breve{K}_m$ and passing to $G_0^{(h)}/G_m^{(h)}$-invariants we obtain that the sequence (18) is exact on the left (resp. on the right) (cf. Theorem A.4 and the analog of (17)).

Assuming $\mathcal{M} = \tilde{M}$ to be Lubin-Tate there is an integer $m \geq 0$ such that the natural map (14) is $G_m^{(h)}$-equivariantly bijective. Putting $M^* := \mathcal{M}^*(\mathcal{X}_0^{(h)}) = \mathrm{Hom}_{A_0^{(h)}}(M, A_0^{(h)})$, there is an isomorphism

$$(A_m^{(h)} \otimes_{A_0^{(h)}} M^*)^{G_m^{(h)}} \simeq \mathrm{Hom}_{\breve{K}_m}((A_m^{(h)} \otimes_{A_0^{(h)}} M)^{G_m^{(h)}}, \breve{K}_m), \tag{19}$$

from which one obtains the isomorphism

$$A_m^{(h)} \otimes_{\breve{K}_m} (A_m^{(h)} \otimes_{A_0^{(h)}} M^*)^{G_m^{(h)}} \simeq A_m^{(h)} \otimes_{A_0^{(h)}} M^*. \tag{20}$$

Using Lemma 3.11 one concludes that the dual bundle $\mathcal{M}^*$ is Lubin-Tate. Further, the analog of (17) leads to a natural isomorphism $\mathbb{D}_{\mathrm{LT}}(\mathcal{M}^*) \simeq \mathbb{D}_{\mathrm{LT}}(\mathcal{M})^*$ of $H_0^{(h)}$-equivariant vector bundles on $\mathcal{Y}_0^{(h)}$. In particular, the category of Drinfeld bundles is closed under duals, too.

The assertions concerning tensor products can be proved as in [20, Theorem 2.13]. The details are left to the reader. □

4 Admissible bundles on the period spaces

In order to extend the equivalence in Theorem 3.7 to objects which are equivariant under the full groups $G^{(h)} = \mathrm{GL}_h(K)$ and $H^{(h)} = (D_K^{(h)})^*$, we need to pass to the corresponding Rapoport-Zink spaces $\underline{\mathcal{X}}_m^{(h)}$ and $\underline{\mathcal{Y}}_m^{(h)}$, i.e. to allow quasi-isogenies of arbitrary heights in the moduli problems of Sections 1 and 2 (cf. [35, Definition 2.15]).

Recall that there are decompositions

$$\underline{\mathcal{X}}_m^{(h)} = \coprod_{n \in \mathbb{Z}} \mathcal{X}_m^{(h),n} \quad \text{and} \quad \underline{\mathcal{Y}}_m^{(h)} = \coprod_{n \in \mathbb{Z}} \mathcal{Y}_m^{(h),n},$$

where $\mathcal{X}_m^{(h),n}$ (resp. $\mathcal{Y}_m^{(h),n}$) is the open subspace on which the universal quasi-isogeny has height nh (resp. n). All spaces $\mathcal{X}_m^{(h),n}$ (resp. $\mathcal{Y}_m^{(h),n}$) are noncanonically isomorphic to $\mathcal{X}_m^{(h),0} = \mathcal{X}_m^{(h)}$ (resp. $\mathcal{Y}_m^{(h),0} = \mathcal{Y}_m^{(h)}$). For any two integers m' and m with $m' \geq m \geq 0$ and for any integer n there are finite étale Galois morphisms $\mathcal{X}_{m'}^{(h),n} \to \mathcal{X}_m^{(h),n}$ and $\mathcal{Y}_{m'}^{(h),n} \to \mathcal{Y}_m^{(h),n}$ which, for $n = 0$, are given by

(4) and (6).

For each integer $m \geq 0$ there is a left action of $H^{(h)}$ on the space $\underline{\mathcal{Y}}_m^{(h)}$ such that $\delta(\mathcal{Y}_m^{(h),n}) = \mathcal{Y}_m^{(h),n-v_D(\delta)}$ for all integers n and all elements $\delta \in H^{(h)}$. It restricts to the action of $H_0^{(h)}$ on $\mathcal{Y}_m^{(h)}$ considered in section 1. Further, the covering morphisms $\underline{\mathcal{Y}}_{m'}^{(h)} \to \underline{\mathcal{Y}}_m^{(h)}$ are $H^{(h)}$-equivariant.

There is also a left action of $G_0^{(h)}$ on $\underline{\mathcal{Y}}_m^{(h)}$, commuting with the action of $H^{(h)}$ and respecting the components $\mathcal{Y}_m^{(h),n}$. For $n = 0$ it is the action considered in section 1. It extends to a left action of $G^{(h)}$ on the family $(\underline{\mathcal{Y}}_m^{(h)})_{m \geq 0}$ in the following sense.

If $U \subseteq G_0^{(h)}$ is an open subgroup then we choose an integer $m \geq 0$ such that $G_m^{(h)} \subseteq U$ and set $\underline{\mathcal{Y}}_U^{(h)} := \underline{\mathcal{Y}}_m^{(h)}/U$ with the induced action of $H^{(h)}$. If $g \in G^{(h)}$ is an element such that $gUg^{-1} \subseteq G_0^{(h)}$ then there is an $H^{(h)}$-equivariant isomorphism

$$g : \underline{\mathcal{Y}}_U^{(h)} \longrightarrow \underline{\mathcal{Y}}_{gUg^{-1}}^{(h)}.$$

If $U \subseteq G_0^{(h)}$ is an open subgroup and if $g_1, g_2 \in G^{(h)}$ are elements such that $g_1 U g_1^{-1} \subseteq G_0^{(h)}$ and $g_2 g_1 U g_1^{-1} g_2^{-1} \subseteq G_0^{(h)}$ then the diagram

$$\begin{tikzcd}
\underline{\mathcal{Y}}_U^{(h)} \arrow[rr, "g_1"] \arrow[dr, "g_2 g_1"'] & & \underline{\mathcal{Y}}_{g_1 U g_1^{-1}}^{(h)} \arrow[dl, "g_2"] \\
& \underline{\mathcal{Y}}_{g_2 g_1 U g_1^{-1} g_2^{-1}}^{(h)} &
\end{tikzcd}$$

commutes.

If U and U' are open subgroups of $G_0^{(h)}$ such that $U' \subseteq U$ then there are morphisms $q_{U',U} : \underline{\mathcal{Y}}_{U'}^{(h)} \to \underline{\mathcal{Y}}_U^{(h)}$. If in particular $U' = G_{m'}^{(h)}$ and $U = G_m^{(h)}$ for integers m' and m with $m' \geq m \geq 0$ then $q_{U',U}$ is the covering morphism (4). Set $q_U := q_{U,G_0^{(h)}}$.

Following [19, Chapitre I, Section IV.11], a *left $H^{(h)}$-equivariant cartesian coherent module on the Lubin-Tate tower* is a family $(\mathcal{N}_U)_{U \subseteq G_0^{(h)}}$ of left $H^{(h)}$-equivariant coherent modules $\mathcal{N}_U$ on $\underline{\mathcal{Y}}_U^{(h)}$ for any open subgroup U of $G_0^{(h)}$ together with $H^{(h)}$-equivariant isomorphisms

$$i_{U',U} : q_{U',U}^*(\mathcal{N}_U) \longrightarrow \mathcal{N}_{U'} \quad \text{and} \quad c_g : (g^{-1})^*(\mathcal{N}_{g^{-1}Ug}) \longrightarrow \mathcal{N}_U$$

for any two open subgroups U' and U of $G_0^{(h)}$ such that $U' \subseteq U$ and for all elements $g \in G^{(h)}$ such that $g^{-1}Ug \subseteq G_0^{(h)}$. These are subject to the obvious cocycle relations.

There is a left action of $H^{(h)}$ on $\mathbb{P}_{\breve{K}}^{h-1}$ and an étale $H^{(h)}$-equivariant rigid analytic morphism

$$\Phi : \underline{\mathcal{Y}}_0^{(h)} \longrightarrow \mathbb{P}_{\breve{K}}^{h-1},$$

the so-called *period morphism*, whose restriction $\Phi_0 := \Phi|\mathcal{Y}_0^{(h)}$ to $\mathcal{Y}_0^{(h)}$ is the morphism constructed in [23, Section 23]. Given a coherent $H^{(h)}$-equivariant module $\mathcal{F}$ on $\mathbb{P}^{h-1}_{\breve{K}}$ consider the family

$$\Phi^*_\infty(\mathcal{F}) := ((\Phi \circ q_U)^*(\mathcal{F}))_{U \subseteq G_0^{(h)}}$$

of $H^{(h)}$-equivariant coherent modules on the Lubin-Tate tower.

Theorem 4.1. *The functor $(\mathcal{F} \mapsto \Phi^*_\infty(\mathcal{F}))$ is an equivalence between the category of $H^{(h)}$-equivariant coherent modules on $\mathbb{P}^{h-1}_{\breve{K}}$ and the category of $H^{(h)}$-equivariant cartesian coherent modules on the Lubin-Tate tower.*

Proof: This can be proved as in [19, Chapitre I, Proposition IV.11.20]. $\square$

Given a vector bundle $\mathcal{M}$ on $\mathcal{X}_0^{(h)}$ (resp. $\mathcal{N}$ on $\mathbb{P}^{h-1}_{\breve{K}}$) which is equivariant with respect to $G^{(h)}$ (resp. $H^{(h)}$) we denote by $\mathrm{res}^{G^{(h)}}_{G_0^{(h)}}\mathcal{M}$ (resp. $\mathrm{res}^{H^{(h)}}_{H_0^{(h)}}\mathcal{N}$) the $G_0^{(h)}$-equivariant vector bundle on $\mathcal{X}_0^{(h)}$ (resp. the $H_0^{(h)}$-equivariant vector bundle on $\mathbb{P}^{h-1}_{\breve{K}}$) obtained by restriction of the action from $G^{(h)}$ to $G_0^{(h)}$ (resp. from $H^{(h)}$ to $H_0^{(h)}$).

Definition 4.2. (i) A $G^{(h)}$-equivariant vector bundle $\mathcal{M} = \tilde{M}$ of finite rank on $\mathcal{X}_0^{(h)}$ is called *Lubin-Tate* if the $G_0^{(h)}$-equivariant vector bundle $\mathrm{res}^{G^{(h)}}_{G_0^{(h)}}\mathcal{M}$ is Lubin-Tate in the sense of Definition 3.4 (i).

(ii) An $H^{(h)}$-equivariant vector bundle of finite rank on $\mathbb{P}^{h-1}_{\breve{K}}$ is called *Drinfeld* if the $H_0^{(h)}$-equivariant vector bundle $\Phi_0^*(\mathrm{res}^{H^{(h)}}_{H_0^{(h)}}\mathcal{F})$ on $\mathcal{Y}_0^{(h)}$ is Drinfeld in the sense of Definition 3.4 (ii).

In order to relate the categories of Lubin-Tate and Drinfeld bundles we need to examine the actions of $G^{(h)}$ and $H^{(h)}$ on the inductive limit of the rings of sections of the spaces $\underline{\mathcal{X}}_m^{(h)}$ and $\underline{\mathcal{Y}}_m^{(h)}$.

For any integer n and any integer $m \geq 0$ we let $A_m^{(h),n} := \mathcal{O}(\mathcal{X}_m^{(h),n})$ and $B_m^{(h),n} := \mathcal{O}(\mathcal{Y}_m^{(h),n})$, so that

$$\mathbb{A}_m^{(h)} := \prod_{n \in \mathbb{Z}} A_m^{(h),n} \quad \text{and} \quad \mathbb{B}_m^{(h)} := \prod_{n \in \mathbb{Z}} B_m^{(h),n}$$

are the rings of global sections of $\underline{\mathcal{X}}_m^{(h)}$ and $\underline{\mathcal{Y}}_m^{(h)}$, respectively. We endow them with the product topology of the $\breve{K}$-algebras $A_m^{(h),n}$ and $B_m^{(h),n}$. The above covering morphisms allow us to define

$$\mathbb{A}_\infty^{(h)} := \varinjlim_m \mathbb{A}_m^{(h)} \quad \text{and} \quad \mathbb{B}_\infty^{(h)} := \varinjlim_m \mathbb{B}_m^{(h)}, \tag{21}$$

endowed with the topology of the locally convex inductive limit in the sense of [36, §5.E]. Further, the actions of $G^{(h)}$ and $H^{(h)}$ give rise to commuting continuous $\breve{K}$-linear left actions of $G^{(h)}$ and $H^{(h)}$ on $\mathbb{A}_\infty^{(h)}$ and $\mathbb{B}_\infty^{(h)}$.

For any integer $m \geq 0$ we let $E_m^{(h),n}$ and $F_m^{(h),n}$ denote the separable closure of $\breve{K}$ in $A_m^{(h),n}$ and $B_m^{(h),n}$, respectively. We know from Proposition 2.7 and the proof of Theorem 1.4 that each of these fields is isomorphic to $\breve{K}_m$, hence is Galois over $\breve{K}$. It follows that the subalgebra $\prod_n E_m^{(h),n}$ (resp. $\prod_n F_m^{(h),n}$) of $\mathbb{A}_m^{(h)}$ (resp. $\mathbb{B}_m^{(h)}$) is stable under the action of $G^{(h)} \times H^{(h)}$. Using Theorem 1.4 and Theorem 2.8 one can show $\prod_n E_m^{(h),n}$ and $\prod_n F_m^{(h),n}$ to be $G^{(h)} \times H^{(h)}$-equivariantly isomorphic.

In order to ease the notation we identify all fields $E_m^{(h),n}$ and $F_m^{(h),n}$ with $\breve{K}_m$. Define the rigid analytic $\breve{K}$-variety

$$\underline{\mathcal{Z}}_m^{(h)} := \coprod_{n\in\mathbb{Z}} \mathcal{X}_m^{(h),n} \times_{\breve{K}_m} \mathcal{Y}_m^{(h),n}$$

and denote by $\mathbb{C}_m^{(h)} := \mathcal{O}(\underline{\mathcal{Z}}_m^{(h)}) = \prod_{n\in\mathbb{Z}} C_m^{(h),n}$ its ring of global sections, endowed with the product topology. Here

$$C_m^{(h),n} := \mathcal{O}(\mathcal{X}_m^{(h),n} \times_{\breve{K}_m} \mathcal{Y}_m^{(h),n}) \simeq A_m^{(h),n} \hat{\otimes}_{\breve{K}_m} B_m^{(h),n}$$

for every integer n. We also set $\mathbb{C}_\infty^{(h)} := \varinjlim_m \mathbb{C}_m^{(h)}$, endowed with the topology of the locally convex inductive limit.

There are $\breve{K}$-linear diagonal actions of $G^{(h)}$ and $H^{(h)}$ on $\mathbb{C}_\infty^{(h)}$ such that for each integer $m \geq 0$ the induced action of $G_0^{(h)} \times H_0^{(h)}$ on $C_m^{(h)}$ coincides with the one considered in Section 3.

Theorem 4.3. *For any integer $m \geq 0$ there are $G_0^{(h)} \times H^{(h)}$-equivariant isomorphisms $(\mathbb{C}_\infty^{(h)})^{G_m^{(h)}} \simeq (\mathbb{B}_\infty^{(h)})^{G_m^{(h)}} \simeq \mathbb{B}_m^{(h)}$. For any integer $m \geq 0$ there are $G^{(h)} \times H^{(h)}$-equivariant isomorphisms $(\mathbb{C}_\infty^{(h)})^{H_m^{(h)}} \simeq (\mathbb{A}_\infty^{(h)})^{H_m^{(h)}} \simeq \mathbb{A}_m^{(h)}$. Further, $(\mathbb{A}_\infty^{(h)})^{G^{(h)}} \simeq (\mathbb{B}_\infty^{(h)})^{G^{(h)}} \simeq (\mathbb{C}_\infty^{(h)})^{G^{(h)}} \simeq (\mathbb{B}_\infty^{(h)})^{H^{(h)}} \simeq \breve{K}$, and there are $G^{(h)}$-equivariant isomorphisms $(\mathbb{A}_\infty^{(h)})^{H^{(h)}} \simeq (\mathbb{C}_\infty^{(h)})^{H^{(h)}} \simeq A_0^{(h)}$.*

Proof: If $m' \geq m$ is an integer, the subrings $\mathbb{A}_{m'}^{(h)}$, $\mathbb{B}_{m'}^{(h)}$ and $\mathbb{C}_{m'}^{(h)}$ of $\mathbb{A}_\infty^{(h)}$, $\mathbb{B}_\infty^{(h)}$ and $\mathbb{C}_\infty^{(h)}$, respectively, are stable under the actions of $G_m^{(h)}$ and $H_m^{(h)}$. The first assertions of the theorem follow from Theorems 1.4, 2.8 and Lemma 3.3 together with the following consideration.

Let n be an integer and let $\delta \in H^{(h)}$ be such that $v_D(\delta) = -n$. The element δ defines $G_0^{(h)}$-equivariant isomorphisms

$$A_{m'}^{(h)} \longrightarrow A_{m'}^{(h),n} \quad \text{and} \quad B_{m'}^{(h)} \longrightarrow B_{m'}^{(h),n},$$

whence $(A_{m'}^{(h),n})^{G_m^{(h)}} = \breve{K}_m$ and $(B_{m'}^{(h),n})^{H_m^{(h)}} \simeq \breve{K}_m$.

Any element $\beta \in (\mathbb{B}_\infty^{(h)})^{H^{(h)}}$ is contained in some subring $\mathbb{B}_m^{(h)}$. It follows that $\beta \in B_m^{(h)}$, embedded into $\mathbb{B}_m^{(h)}$ via the direct product of the integral powers of some uniformizer of D. Since the projection $\mathbb{B}_m^{(h)} \to B_m^{(h)}$ is $H_0^{(h)}$-equivariant, we obtain $\beta \in \breve{K}$ from Theorem 1.4. The $H^{(h)}$-invariants of $\mathbb{A}_\infty^{(h)}$ and $\mathbb{C}_\infty^{(h)}$ can be computed by the same strategy, using Theorems 2.8 and 3.2.

As seen above, we have $(\mathbb{A}_m^{(h)})^{G_m^{(h)}} = \prod_{n\in\mathbb{Z}} \breve{K}_m$, and this algebra is stable under the action of $G^{(h)}$ with $\mathrm{SL}_h(K)$ acting trivially. Embedding $\breve{K}_m$ diagonally via some element whose determinant has valuation 1, we deduce as above that $(\mathbb{A}_\infty^{(h)})^{G^{(h)}} = \breve{K}$ by using Theorem 2.8.

Finally, any of the subrings $\mathbb{B}_m^{(h)}$ (resp. $\mathbb{C}_m^{(h)}$) of $\mathbb{B}_\infty^{(h)}$ (resp. $\mathbb{C}_\infty^{(h)}$) is stable under the action of $G_0^{(h)}$, so that $(\mathbb{C}_\infty^{(h)})^{G^{(h)}} \simeq (\mathbb{B}_\infty^{(h)})^{G^{(h)}} \subseteq \mathbb{B}_0^{(h)}$. Let $f \in \mathbb{B}_0^{(h)} \subset \mathbb{B}_\infty^{(h)}$ be invariant under $G^{(h)}$. Choosing $g \in G^{(h)}$ with $v_K(\det(g)) = 1$, we see that f is determined by its restriction to the space $\mathcal{Y}_0^{(h)} = \mathcal{Y}_0^{(h),0}$.

According to the proof of [23, Corollary 23.21], there are closed rigid analytic polydiscs $D_1, \ldots, D_n$ in $\underline{\mathcal{Y}}_0^{(h)}$ such that the restrictions $\Phi : D_i \to \Phi(D_i)$ of the period morphism are isomorphisms and such that the subsets $\Phi(D_i)$ form an affinoid covering of $\mathbb{P}_{\breve{K}}^{h-1}$.

We obtain sections $f_i \in \mathcal{O}(\Phi(D_i))$ via $\Phi^*(f_i) = f|D_i$ and claim that f_i and f_j coincide on $\Phi(D_i)\cap\Phi(D_j)$ for any two indices i and j. Since $\mathbb{P}_{\breve{K}}^{h-1}$ is reduced, this can be checked pointwise. If $y_i \in D_i$ and $y_j \in D_j$ are such that $\Phi(y_i) = \Phi(y_j)$ then there is an element $g \in G^{(h)}$, an integer $m \geq 0$ and a point y_i' in $\underline{\mathcal{Y}}_m^{(h)}$ lying above y_i such that $g : \underline{\mathcal{Y}}_m^{(h)} \to \underline{\mathcal{Y}}_0^{(h)}$ is defined and maps y_i' to y_j (cf. [23, p. 82], [35, Section 5.50], or [42, Proposition 2.6.7]). Since the compositions with Φ of the morphism g and the covering morphism $\underline{\mathcal{Y}}_m^{(h)} \to \underline{\mathcal{Y}}_0^{(h)}$ agree, there is a commutative diagram

$$\begin{tikzcd}
\kappa(\Phi(y_i)) = \kappa(\Phi(y_j)) \arrow[r] \arrow[d] & \kappa(y_i) \arrow[d] \\
\kappa(y_j) \arrow[r, "g"'] & \kappa(y_i').
\end{tikzcd}$$

But then $f_i(\Phi(y_i)) = f_j(\Phi(y_j))$ by the $G^{(h)}$-invariance of f.

Therefore, the family $(f_1, \ldots, f_n)$ gives rise to a global section on $\mathbb{P}_{\breve{K}}^{h-1}$. Since $\mathcal{O}(\mathbb{P}_{\breve{K}}^{h-1}) = \breve{K}$ we obtain $f|D_1 \in \breve{K}$ and then $f \in \breve{K}$ since $\mathcal{Y}_0^{(h)}$ is normal and connected (cf. Theorem 1.2 and [10, Lemma 2.1.4]). $\square$

Given a $G^{(h)}$-equivariant Lubin-Tate bundle $\mathcal{M} = \tilde{M}$ on $\mathcal{X}_0^{(h)}$, we claim that the natural map

$$\mathbb{C}_\infty^{(h)} \otimes_{\mathbb{B}_0^{(h)}} (\mathbb{C}_\infty^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}} \longrightarrow \mathbb{C}_\infty^{(h)} \otimes_{A_0^{(h)}} M \tag{22}$$

is bijective. Note first that, as in the proof of Theorem 4.3, $A_0^{(h)}$ is identified with the ring of $H^{(h)}$-invariants of $\mathbb{C}_\infty^{(h)}$ by embedding it diagonally via the integral powers of a fixed element $\delta \in D$ with valuation -1. Choose an integer $m \geq 0$ so that the map (9) is bijective. For any integer n we then have

$$(C_m^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}} \simeq (C_m^{(h),n} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}}$$

via $\delta^n \otimes \mathrm{id}_M$. Since M is finitely presented over $A_0^{(h)}$ (cf. Proposition A.2), the natural $G_0^{(h)}$-equivariant map

$$\mathbb{C}_m^{(h)} \otimes_{A_0^{(h)}} M \longrightarrow \prod_{n\in\mathbb{Z}} (C_m^{(h),n} \otimes_{A_0^{(h)}} M)$$

is bijective (cf. [3, Exercice I.2.9, p. 62]). Since the module $(C_m^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}}$ is finitely presented over $B_0^{(h)}$ (cf. the discussion following Definition 3.4), the same reference together with the above reasoning implies that the natural map

$$\mathbb{B}_0^{(h)} \otimes_{B_0^{(h)}} (C_m^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}} \longrightarrow (\mathbb{C}_m^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}}$$

is bijective. Therefore,

$$\mathbb{C}_m^{(h)} \otimes_{\mathbb{B}_0^{(h)}} (\mathbb{C}_m^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}} \simeq \mathbb{C}_m^{(h)} \otimes_{B_0^{(h)}} (C_m^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}},$$

where $B_0^{(h)}$ is embedded into $\mathbb{C}_m^{(h)}$ via the integral powers of δ. Thus, by the base extension of the isomorphism (9) from $C_m^{(h)}$ to $\mathbb{C}_m^{(h)}$ via the integral powers of δ, we obtain that the natural map

$$\mathbb{C}_m^{(h)} \otimes_{\mathbb{B}_0^{(h)}} (\mathbb{C}_m^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}} \longrightarrow \mathbb{C}_m^{(h)} \otimes_{A_0^{(h)}} M$$

is bijective. Passing to the direct limit over all integers $m' \geq m$, and using Lemma 3.5, we obtain the desired bijectivity of (22).

We obtain the $H^{(h)}$-equivariant vector bundle $\mathcal{N}_{G_0^{(h)}} := [(\mathbb{C}_\infty^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}}]^\sim$ of finite rank on $\underline{\mathcal{Y}}_0^{(h)}$ such that $\mathcal{N}_{G_0^{(h)}} | \mathcal{Y}_0^{(h),n} \simeq [(C_m^{(h),n} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}}]^\sim$ for some integer $m \geq 0$ and any integer n.

The bijection (22) is $G_m^{(h)}$-equivariant with respect to the diagonal action on both sides. Since $(\mathbb{C}_\infty^{(h)} \otimes_{A_0^{(h)}} M)^{G_0^{(h)}}$ is a projective $\mathbb{B}_0^{(h)}$-module, passing to $G_m^{(h)}$-invariants and tensoring with $\mathbb{C}_\infty^{(h)}$ over $\mathbb{B}_m^{(h)}$ shows that the natural map

$$\mathbb{C}_\infty^{(h)} \otimes_{\mathbb{B}_m^{(h)}} (\mathbb{C}_\infty^{(h)} \otimes_{A_0^{(h)}} M)^{G_m^{(h)}} \longrightarrow \mathbb{C}_\infty^{(h)} \otimes_{A_0^{(h)}} M$$

is bijective for any integer $m \geq 0$. We obtain an $H^{(h)}$-equivariant vector bundle $\mathcal{N}_{G_m^{(h)}} := [(\mathbb{C}_\infty^{(h)} \otimes_{A_0^{(h)}} M)^{G_m^{(h)}}]^\sim$ of finite rank on $\underline{\mathcal{Y}}_m^{(h)}$ satisfying $q_{G_m^{(h)}}^*(\mathcal{N}_{G_0^{(h)}}) \simeq \mathcal{N}_{G_m^{(h)}}$.

Given a compact open subgroup U of $G_0^{(h)}$ set $\mathcal{N}_U := q_U^*(\mathcal{N}_{G_0^{(h)}})$, which is an $H^{(h)}$-equivariant vector bundle of finite rank on $\underline{\mathcal{Y}}_U^{(h)}$ with global sections $(\mathbb{C}_\infty^{(h)} \otimes_{A_0^{(h)}} M)^U$.

The diagonal action of $G^{(h)}$ on $\mathbb{C}_\infty^{(h)} \otimes_{A_0^{(h)}} M$ induces on the family $(\mathcal{N}_U)_U$ the structure of an $H^{(h)}$-equivariant cartesian module on the Lubin-Tate tower, in which all modules $\mathcal{N}_U$ are locally free of finite rank. According to Theorem

4.1 it corresponds to an $H^{(h)}$-equivariant vector bundle of finite rank on $\mathbb{P}_{\breve{K}}^{h-1}$ which, by abuse of notation, we denote by $\mathbb{D}_{\mathrm{LT}}(\mathcal{M})$. By construction there is an $H_0^{(h)}$-equivariant isomorphism

$$\Phi_0^*(\operatorname{res}_{H_0^{(h)}}^{H^{(h)}} \mathbb{D}_{\mathrm{LT}}(\mathcal{M})) \simeq \mathbb{D}_{\mathrm{LT}}(\operatorname{res}_{G_0^{(h)}}^{G^{(h)}} \mathcal{M}). \tag{23}$$

Conversely, if $\mathcal{F}$ is an $H^{(h)}$-equivariant Drinfeld bundle on $\mathbb{P}_{\breve{K}}^{h-1}$ then the natural map

$$\mathbb{C}_\infty^{(h)} \otimes_{A_0^{(h)}} (\mathbb{C}_\infty^{(h)} \otimes_{\mathbb{B}_0^{(h)}} \Phi^*(\mathcal{F})(\underline{\mathcal{Y}}_0^{(h)}))^{H^{(h)}} \longrightarrow \mathbb{C}_\infty^{(h)} \otimes_{\mathbb{B}_0^{(h)}} \Phi^*(\mathcal{F})(\underline{\mathcal{Y}}_0^{(h)}) \tag{24}$$

is an isomorphism. Indeed, the $\mathbb{B}_0^{(h)}$-module $\Phi^*(\mathcal{F})(\underline{\mathcal{Y}}_0^{(h)})$ is finitely presented so that there is an $H^{(h)}$-equivariant isomorphism

$$\mathbb{C}_m^{(h)} \otimes_{\mathbb{B}_0^{(h)}} \Phi^*(\mathcal{F})(\underline{\mathcal{Y}}_0^{(h)}) \simeq \prod_{n\in\mathbb{Z}} C_m^{(h),n} \otimes_{B_m^{(h),n}} \Phi^*(\mathcal{F})(\mathcal{Y}_0^{(h),n})$$

for any integer $m \geq 0$. The embedding of $C_m^{(h)} \otimes_{B_0^{(h)}} \Phi^*(\mathcal{F})(\mathcal{Y}_0^{(h)})$ into the right-hand side via the integral powers of a uniformizer of D induces an isomorphism

$$(C_m^{(h)} \otimes_{B_0^{(h)}} \Phi^*(\mathcal{F})(\mathcal{Y}_0^{(h)}))^{H_0^{(h)}} \simeq (\mathbb{C}_m^{(h)} \otimes_{\mathbb{B}_0^{(h)}} \Phi^*(\mathcal{F})(\underline{\mathcal{Y}}_0^{(h)}))^{H^{(h)}}$$

for any integer $m \geq 0$. The bijectivity of (24) follows from Lemma 3.5 and the fact that $\Phi^*(\mathcal{F})|\,\mathcal{Y}_0^{(h)} = \Phi_0^*(\mathcal{F})$ is Drinfeld in the sense of Definition 3.4. As a consequence,

$$\mathbb{D}_{\mathrm{Dr}}(\mathcal{F}) := [(\mathbb{C}_\infty^{(h)} \otimes_{\mathbb{B}_0^{(h)}} \Phi^*(\mathcal{F})(\underline{\mathcal{Y}}_0^{(h)}))^{H^{(h)}}]^\sim$$

is a vector bundle of finite rank on $\mathcal{X}_0^{(h)}$. Further, the action of $G^{(h)}$ on $\mathbb{C}_\infty^{(h)}$ induces on $\mathbb{D}_{\mathrm{Dr}}(\mathcal{F})$ the structure of a $G^{(h)}$-equivariant vector bundle. By construction there is a $G_0^{(h)}$-equivariant isomorphism

$$\operatorname{res}_{G_0^{(h)}}^{G^{(h)}} \mathbb{D}_{\mathrm{Dr}}(\mathcal{F}) \simeq \mathbb{D}_{\mathrm{Dr}}(\Phi_0^*(\operatorname{res}_{H_0^{(h)}}^{H^{(h)}} \mathcal{F})). \tag{25}$$

As in Section 3 we denote by $\mathcal{B}_{\mathcal{X}_0^{(h)}}(G^{(h)})$ and $\mathcal{B}_{\mathbb{P}_{\breve{K}}^{h-1}}(H^{(h)})$ the category of $G^{(h)}$-equivariant vector bundles of finite rank on $\mathcal{X}_0^{(h)}$ and the category of $H^{(h)}$-equivariant vector bundles of finite rank on $\mathbb{P}_{\breve{K}}^{h-1}$, respectively. The full subcategories consisting of Lubin-Tate and Drinfeld bundles are marked with a corresponding superscript. The following result is a direct consequence of Theorem 3.7, Theorem 4.1, (23) and (25).

Theorem 4.4. *If $\mathcal{M}$ is a $G^{(h)}$-equivariant Lubin-Tate bundle on $\mathcal{X}_0^{(h)}$ then $\mathbb{D}_{\mathrm{LT}}(\mathcal{M})$ is an $H^{(h)}$-equivariant Drinfeld bundle on $\mathbb{P}_{\breve{K}}^{h-1}$. If $\mathcal{F}$ is an $H^{(h)}$-equivariant Drinfeld bundle on $\mathbb{P}_{\breve{K}}^{h-1}$ then $\mathbb{D}_{\mathrm{Dr}}(\mathcal{F})$ is a $G^{(h)}$-equivariant Lubin-Tate bundle on $\mathcal{X}_0^{(h)}$. The assignments*

$$\mathbb{D}_{\mathrm{LT}} := (\mathcal{M} \mapsto \mathbb{D}_{\mathrm{LT}}(\mathcal{M})) \quad \textit{and} \quad \mathbb{D}_{\mathrm{Dr}} := (\mathcal{F} \mapsto \mathbb{D}_{\mathrm{Dr}}(\mathcal{F}))$$

are mutually quasi-inverse equivalences of categories between $\mathcal{B}^{\mathrm{LT}}_{\mathcal{X}_0^{(h)}}(G^{(h)})$ and $\mathcal{B}^{\mathrm{Dr}}_{\mathbb{P}_{\breve{K}}^{h-1}}(H^{(h)})$. □

As for the formal properties of the categories of Lubin-Tate and Drinfeld bundles on $\mathcal{X}_0^{(h)}$ and $\mathbb{P}_{\breve{K}}^{h-1}$, Theorem 3.12 has an exact analog which we refrain from repeating.

Let $\mathrm{Rep}_{\breve{K}}(H^{(h)})$ denote the category of finite dimensional representations of $H^{(h)}$ over $\breve{K}$. Given an object ρ of this category set $\mathcal{F}(\rho) := \mathcal{O}_{\mathbb{P}_{\breve{K}}^{h-1}} \otimes_{\breve{K}} \rho$, which is an $H^{(h)}$-equivariant vector bundle of finite rank on $\mathbb{P}_{\breve{K}}^{h-1}$. Since $\mathcal{F}(\rho)(\mathbb{P}_{\breve{K}}^{h-1}) \simeq \rho$ as an $H^{(h)}$-representation over $\breve{K}$, the functor $(\rho \mapsto \mathcal{F}(\rho))$ from $\mathrm{Rep}_{\breve{K}}(H^{(h)})$ to $\mathcal{B}_{\mathbb{P}_{\breve{K}}^{h-1}}(H^{(h)})$ is an embedding of categories.

We say that a finite dimensional representation ρ of $H^{(h)}$ over $\breve{K}$ is *Drinfeld* if the $H^{(h)}$-equivariant vector bundle $\mathcal{F}(\rho)$ on $\mathbb{P}_{\breve{K}}^{h-1}$ is Drinfeld in the sense of Definition 4.2. In this case we set $\mathbb{D}_{\mathrm{Dr}}(\rho) := \mathbb{D}_{\mathrm{Dr}}(\mathcal{F}(\rho))$ and have isomorphisms

$$(26) \quad \mathbb{D}_{\mathrm{Dr}}(\rho) \simeq [(\mathbb{C}_\infty^{(h)} \otimes_{\breve{K}} \rho)^{H^{(h)}}]^\sim \quad \text{and} \quad \rho \simeq (\mathbb{C}_\infty^{(h)} \otimes_{A_0^{(h)}} \mathbb{D}_{\mathrm{Dr}}(\rho)(\mathcal{X}_0^{(h)}))^{G^{(h)}}$$

in $\mathcal{B}_{\mathcal{X}_0^{(h)}}(G^{(h)})$ and $\mathrm{Rep}_{\breve{K}}(H^{(h)})$, respectively.

Given a finite dimensional representation V of $G^{(h)}$ over $\breve{K}$ we shall also consider the $G^{(h)}$-equivariant vector bundle $\mathcal{M}(V) := (A_0^{(h)} \otimes_{\breve{K}} V)^\sim$ on $\mathcal{X}_0^{(h)}$.

Theorem 4.5. *Any finite dimensional smooth representation ρ of $H^{(h)}$ over $\breve{K}$ is Drinfeld. If V is a finite dimensional smooth representation of $G^{(h)}$ over $\breve{K}$ then the $G^{(h)}$-equivariant vector bundle $\mathcal{M}(V)$ on $\mathcal{X}_0^{(h)}$ is Lubin-Tate.*

Proof: This follows from (23), (25) and Theorem 3.8. □

Note that by Remark 3.9 the essential images of the functors $\mathbb{D}_{\mathrm{LT}}(\mathcal{M}(\,\cdot\,))$ and $\mathcal{F}(\,\cdot\,)$ do not seem to agree.

Proposition 4.6. *Identifying the category of $G^{(1)}$- and $H^{(1)}$-equivariant vector bundles on $\mathcal{X}_0^{(1)} = \mathrm{Sp}(\breve{K})$ and $\mathbb{P}_{\breve{K}}^0 = \mathrm{Sp}(\breve{K})$ with the category of finite dimensional representations of $G^{(1)}$ and $H^{(1)}$ over $\breve{K}$, respectively, we have $\mathcal{B}^{\mathrm{LT}}_{\mathcal{X}_0^{(1)}}(G^{(1)}) = \mathrm{Rep}^\infty_{\breve{K}}(G^{(1)})$ and $\mathcal{B}^{\mathrm{Dr}}_{\mathbb{P}^0_{\breve{K}}}(H^{(1)}) = \mathrm{Rep}^\infty_{\breve{K}}(H^{(1)})$. Identifying $G^{(1)}$ and $H^{(1)}$ with K^* we have*

$$\mathrm{tr}_{\mathbb{D}_{\mathrm{Dr}}(\rho)}(\alpha) = \mathrm{tr}_\rho(\alpha)$$

for any finite dimensional smooth representation ρ of K^* over $\breve{K}$ and any element $\alpha \in K^*$.

Proof: In this case we have $A_m^{(1)} = B_m^{(1)} = C_m^{(1)} = \breve{K}_m$ for all integers $m \geq 0$. According to Lemma 3.11, the restriction of any Lubin-Tate (resp. Drinfeld) representation of $G^{(1)}$ (resp. $H^{(1)}$) to some suitable subgroup $G_m^{(1)}$ (resp. $H_m^{(1)}$) is trivial. The converse is the content of Theorem 4.5.

If ρ is a finite dimensional smooth representation of $H^{(1)}$ over $\breve{K}$, consider the isomorphism

$$\mathbb{C}_\infty^{(1)} \otimes_{\breve{K}} \mathbb{D}_{\mathrm{Dr}}(\rho) \simeq \mathbb{C}_\infty^{(1)} \otimes_{\breve{K}} \rho,$$

which is checked to be K^*-equivariant with respect to the $\mathbb{C}_\infty^{(1)}$-linear extension of the action of K^* on ρ and $\mathbb{D}_{\mathrm{Dr}}(\rho)$, respectively. $\square$

The compatibility with traces in Proposition 4.6 extends to a more general situation. For this, let $g \in G^{(h)}$ be regular elliptic, i.e. assume its minimal polynomial $\mu_g(t) \in K[t]$ to be irreducible and separable of degree h. Denote by $L := K[g] \simeq K[t]/(\mu_g(t))$ the subfield of $\mathrm{M}_h(K)$ generated by g. Let C denote the completion of an algebraic closure of K. The fixed points x_i of g in $\mathbb{P}_K^{h-1}(C)$ correspond bijectively to the eigenspaces of g in C^h, hence to the h distinct roots $\alpha_1, \ldots, \alpha_h$ of μ_g in C. We have $\kappa(x_i) \simeq K[\alpha_i] \simeq L$ for all indices i, and all points x_i are contained in $\Omega_K^{(h)}(C)$.

Note also that there is a K-linear isomorphism of fields $\tau : C \to C$ with $\tau(\alpha_i) = \alpha_j$ for any two indices i and j. Since g is K-linear we have $\tau(x_i) = x_j$. Thus, all points $x_1, \ldots, x_h$ are conjugate over K and have the same underlying image in $\Omega_K^{(h)}$.

Now let $\mathcal{M}$ be a $G^{(h)}$-equivariant vector bundle of finite rank on $\mathcal{X}_0^{(h)} \simeq \Omega_K^{(h)} \times_K \breve{K}$. For any point $x \in \mathcal{X}_0^{(h)}(C)$ the reduction $\mathcal{M} \otimes \kappa(x)$ of $\mathcal{M}$ at x is a finite dimensional $\kappa(x)$-linear representation of the stabilizer subgroup $G_x^{(h)}$ of $G^{(h)}$ at x. If $g \in G_x^{(h)}$ we denote by $\mathrm{tr}_{\mathcal{M}\otimes\kappa(x)}(g) \in \kappa(x)$ the trace of g on $\mathcal{M} \otimes \kappa(x)$.

Recall that there is a bijection between the set of conjugacy classes of regular elliptic elements in $G^{(h)}$ and the set of conjugacy classes of certain elements in $H^{(h)}$. It is characterized by the identity of the corresponding minimal polynomials over K.

Theorem 4.7. *Let $h \geq 1$ be an integer, let $g \in G^{(h)}$ be regular elliptic and let $\delta \in H^{(h)}$ be a representative of the conjugacy class corresponding to the conjugacy class of g in $G^{(h)}$. If ρ is a finite dimensional smooth representation of $H^{(h)}$ over $\breve{K}$ then*

$$\mathrm{tr}_{\mathbb{D}_{\mathrm{Dr}}(\rho)\otimes\kappa(x)}(g) = \mathrm{tr}_\rho(\delta) \tag{27}$$

for any fixed point $x \in \mathcal{X}_0^{(h)}(C)$ of g.

Proof: Choose a representative $\delta \in D$ of the conjugacy class corresponding to g and consider the subfield $L := K[\delta]$ of D. Since L is of dimension h over K we shall construct in Section 5 an embedding $L \hookrightarrow \mathrm{M}_h(K)$ and an L^*-equivariant morphism $\mathrm{i_{L|K}} : \mathcal{X}_{em,L}^{(1)} \to \mathcal{X}_{m,K}^{(h)}$ for any integer $m \geq 0$. Here $e = e_{L|K}$ denotes the ramification index of L over K. For $m = 0$, the morphism $\mathrm{i_{L|K}}$ defines a fixed point x' of the image g' of δ in $G^{(h)}$ under the embedding $L^* \hookrightarrow G^{(h)}$.

According to the base change property exhibited in Theorem 5.3, there is an L^*-equivariant isomorphism

$$(\mathbb{D}_{\mathrm{Dr}}(\rho) \otimes \kappa(x')) \otimes_{\kappa(x')} \breve{L} \simeq \mathbb{D}_{\mathrm{Dr}}(\mathrm{res}_{L^*}^{H^{(h)}}(\breve{L} \otimes_{\breve{K}} \rho)).$$

Thus, Proposition 4.6 implies that

$$\mathrm{tr}_{\mathbb{D}_{\mathrm{Dr}}(\rho)\otimes\kappa(x')}(g') = \mathrm{tr}_\rho(\delta). \tag{28}$$

According to the theorem of Skolem-Noether (cf. [2, VIII.10.1 Théorème 1]) there is an element $\gamma \in G^{(h)}$ such that $\gamma g' \gamma^{-1} = g$. Setting $x := \gamma \cdot x'$, the element γ induces an isomorphism of fields $\gamma : \kappa(x) \to \kappa(x')$ and a $\breve{K}$-linear bijection

$$\mathbb{D}_{\mathrm{Dr}}(\rho) \otimes \kappa(x) \longrightarrow \mathbb{D}_{\mathrm{Dr}}(\rho) \otimes \kappa(x'),$$

compatible with γ. Thus, x is a fixed point of g and $\gamma(\mathrm{tr}_{\mathbb{D}_{\mathrm{Dr}}(\rho)\otimes\kappa(x)}(g)) = \mathrm{tr}_{\mathbb{D}_{\mathrm{Dr}}(\rho)\otimes\kappa(x')}(g')$. It follows from (28) and the fact that $\mathrm{tr}_\rho(\delta)$ lies in $\breve{K}$ that

$$\mathrm{tr}_{\mathbb{D}_{\mathrm{Dr}}(\rho)\otimes\kappa(x)}(g) = \mathrm{tr}_\rho(\delta).$$

The element γ induces a bijection between the fixed points of g' and those of g. By the arguments just given we may assume $g = g'$ and $x = x'$.

Let $x'' \in \mathcal{X}_0^{(h)}(C)$ be a second fixed point of g. According to our preliminary remarks the points x and x'' are conjugate over K. Thus, there is a K-linear isomorphism $\tau : \breve{K} \to \breve{K}$ of fields such that x'' is the image of the L^*-equivariant morphism

$$\mathcal{X}_{0,L}^{(1),(\tau)} \xrightarrow{\mathrm{i}_{L|K}^{(\tau)}} \mathcal{X}_{0,K}^{(h),(\tau)} \longrightarrow \mathcal{X}_{0,K}^{(h)}$$

of $\breve{K}$-varieties. Here $Z^{(\tau)} := Z \times_{\breve{K}} \breve{K}^{(\tau)}$ denotes the base extension along τ for any rigid $\breve{K}$-variety Z, and $Z^{(\tau)} \to Z$ is the natural projection. Denote by $\tilde{\tau} : \kappa(x'') \to \breve{L}^{(\tau)}$ the induced homomorphism of fields over $\breve{K}$. Using the embedding $\tau : \breve{K} \to \breve{K} \to \breve{L}$ in the construction of the morphism $\mathrm{i_{L|K}}$ of section 5, we obtain precisely the morphism $\mathrm{i}_{L|K}^{(\tau)}$. Therefore,

$$(\mathbb{D}_{\mathrm{Dr}}(\rho) \otimes \kappa(x'')) \otimes_{\kappa(x'')} \breve{L}^{(\tau)} \simeq \mathbb{D}_{\mathrm{Dr}}(\mathrm{res}_{L^*}^{H^{(h)}}(\breve{L}^{(\tau)} \otimes_{\breve{K}} \rho))$$

and $\tilde{\tau}(\mathrm{tr}_{\mathbb{D}_{\mathrm{Dr}}(\rho)\otimes\kappa(x'')}(g)) = \tau(\mathrm{tr}_\rho(\delta))$, as above. Since the restriction of $\tilde{\tau}$ to the subfield $\breve{K}$ of $\kappa(x'')$ coincides with τ, the latter equation implies that $\mathrm{tr}_{\mathbb{D}_{\mathrm{Dr}}(\rho)\otimes\kappa(x'')}(g) = \mathrm{tr}_\rho(\delta)$. □

Remark 4.8. The space of global sections of a $G^{(h)}$-equivariant vector bundle $\mathcal{M}$ of finite rank on $\mathcal{X}_0^{(h)}$ is a $\breve{K}$-Fréchet space with an action of $G^{(h)}$ by continuous $\breve{K}$-linear automorphisms. This construction gives rise to many interesting examples of *locally analytic representations* in the sense of [39, Section 3] (cf. [38] and [33]).

A first attempt to define the trace of a locally analytic representation, at least in special cases, was made by Diepholz in [13]. It is not clear if it covers our situation. If $g \in G^{(h)}$ is a regular elliptic element one might alternatively choose an embedding $\kappa(x) \to C$ for any fixed point $x \in \mathcal{X}_0^{(h)}(C)$ of g. Assuming the integer h to be prime to the characteristic of K we set

$$\mathrm{tr}_{\mathcal{M}}(g) := \frac{1}{h} \sum_{\substack{x \in \mathcal{X}_0^{(h)}(C) \\ g \cdot x = x}} \mathrm{tr}_{\mathcal{M}\otimes\kappa(x)}(g) \in C,$$

and call $\mathrm{tr}_{\mathcal{M}}(g)$ the *trace of* g *on* $\mathcal{M}$. With this convention Theorem 4.7 implies the more suggestive formula

$$\mathrm{tr}_{\mathbb{D}_{\mathrm{Dr}}(\rho)}(g) = \mathrm{tr}_\rho(\delta)$$

for any finite dimensional smooth representation ρ of $H^{(h)}$ over $\breve{K}$.

This compatibility with traces raises the question of how the restriction of the equivalence in Theorem 4.4 to finite dimensional smooth representations of $H^{(h)}$ over $\breve{K}$ is related to the Jacquet-Langlands correspondence.

If ρ is a finite dimensional smooth representation of $H^{(h)}$ over $\breve{K}$, the naive approach of considering the subspace of $G^{(h)}$-smooth vectors in $\mathbb{D}_{\mathrm{Dr}}(\rho)$ or in its continuous $\breve{K}$-linear dual does not yield anything useful. If for example $\rho = \breve{K}$ is the trivial representation of $H^{(h)}$, then $\mathcal{O}(\mathbb{D}_{\mathrm{Dr}}(\breve{K})) = \mathcal{O}(\mathcal{X}_0^{(h)}) = A_0^{(h)}$, by Theorem 4.3. It follows from Lemma 2.9 that the subspace of $G^{(h)}$-smooth vectors of the latter is just $\breve{K}$, i.e. the trivial representation of $G^{(h)}$.

On the other hand, the bundles $\mathbb{D}_{\mathrm{Dr}}(\rho)$ carry an additional piece of structure. Namely, they come equipped with a $G^{(h)}$-equivariant integrable connection. The corresponding de Rham complex is simply the ρ-isotypic component of the de Rham complex of $\mathcal{O}_{\mathcal{X}_m^{(h)}}$ for m sufficiently large. It is tempting to wonder about the connection between the smooth $G^{(h)}$-representation associated to ρ via the Jacquet-Langlands correspondence and the de Rham cohomology of $\mathbb{D}_{\mathrm{Dr}}(\rho)$. For example, the smooth $G^{(h)}$-representation corresponding to the trivial representation $\rho = \breve{K}$ is the Steinberg representation. According to a theorem of Schneider and Stuhler, the latter also coincides with the continuous $\breve{K}$-linear dual of $\mathrm{H}_{\mathrm{dR}}^{h-1}(\mathcal{X}_0^{(h)}, \mathcal{O}_{\mathcal{X}_0^{(h)}}) = \mathrm{H}_{\mathrm{dR}}^{h-1}(\mathcal{X}_0^{(h)}, \mathbb{D}_{\mathrm{Dr}}(\breve{K}))$ (cf. [37, §3 Theorem 1 and §4 Lemma 1]).

5 Functoriality

Let L be a field extension of K which is of finite degree, and denote by $n := [L : K]$, $e = e_{L|K}$ and $f = f_{L|K}$ its degree, its ramification index and its residue class degree, respectively. All objects of the previous sections will be marked with an additional index L or K, according to which base field they refer to.

Fix an embedding $\breve{K} \subseteq \breve{L}$ over K and consider the induced embedding $\breve{\mathfrak{o}}_K \subseteq \breve{\mathfrak{o}}_L$. Since the residue class fields of $\breve{\mathfrak{o}}_K$ and $\breve{\mathfrak{o}}_L$ coincide, restriction of scalars defines an embedding $\mathrm{res}_{L|K} : \mathcal{C}_L \to \mathcal{C}_K$. If $h \geq 1$ and $m \geq 0$ are integers then the restriction $\mathfrak{Y}_{m,K}^{(h)} \circ \mathrm{res}_{L|K}$ of the set valued functor $\mathfrak{Y}_{m,K}^{(h)}$ to the subcategory $\mathcal{C}_L$ of $\mathcal{C}_K$ is represented by the formal scheme $\mathrm{Spf}(R_{m,K}^{(h)} \hat{\otimes}_{\breve{\mathfrak{o}}_K} \breve{\mathfrak{o}}_L) = \mathfrak{Y}_{m,K}^{(h)} \times_{\breve{\mathfrak{o}}_K} \breve{\mathfrak{o}}_L$.

Fix an integer $h \geq 1$. Via restriction of scalars, the one dimensional formal $\mathfrak{o}_L$-module $\mathbb{H}_L^{(h)}$ of height h over k^s is a one dimensional formal $\mathfrak{o}_K$-module of height nh. In particular, there is an isomorphism $\mathbb{H}_L^{(h)} \simeq \mathbb{H}_K^{(nh)}$ (cf. [14, Proposition 1.7]), giving rise to an embedding of rings

$$\mathfrak{o}_{D_L^{(h)}} \simeq \mathrm{End}_{\mathfrak{o}_L}(\mathbb{H}_L^{(h)}) \subseteq \mathrm{End}_{\mathfrak{o}_K}(\mathbb{H}_L^{(h)}) \simeq \mathrm{End}_{\mathfrak{o}_K}(\mathbb{H}_K^{(nh)}) \simeq \mathfrak{o}_{D_K^{(nh)}}. \tag{29}$$

For any integer $m \geq 0$ we define a natural transformation

$$\mathrm{r}_{\mathrm{L|K}} : \mathfrak{Y}_{em,L}^{(h)} \longrightarrow \mathfrak{Y}_{m,K}^{(nh)} \circ \mathrm{res}_{L|K} \tag{30}$$

of set valued functors on $\mathcal{C}_L$ as follows.

Given an object $R = (R, \mathfrak{m}_R)$ of $\mathcal{C}_L$ and an isomorphism class in $\mathfrak{Y}^{(h)}_{em,L}(R)$, represented by a triple (H, ρ, φ), define its image under $\mathrm{r}_{\mathrm{L|K}}(R)$ to be the isomorphism class in $\mathfrak{Y}^{(nh)}_{m,K}(R)$ represented by the triple (H', ρ', φ') where H' is obtained from H via restriction of scalars. Further, ρ' is the composition of ρ with the fixed isomorphism $\mathbb{H}^{(h)}_L \simeq \mathbb{H}^{(nh)}_K$. Finally, choosing an $\mathfrak{o}_K$-linear isomorphism $\mathfrak{o}_L \simeq \mathfrak{o}_K^n$, we obtain an $\mathfrak{o}_K$-linear isomorphism

$$(\pi_L^{-em}\mathfrak{o}_L/\mathfrak{o}_L)^h = (\pi_K^{-m}\mathfrak{o}_L/\mathfrak{o}_L)^h \xrightarrow{\sim} (\pi_K^{-m}\mathfrak{o}_K/\mathfrak{o}_K)^{nh}$$

and define φ' to be the composition of its inverse with φ.

The identification $\mathfrak{o}_L \simeq \mathfrak{o}_K^n$ defines an embedding

$$G^{(h)}_{0,L} \hookrightarrow G^{(nh)}_{0,K}. \tag{31}$$

Letting $G^{(h)}_{0,L}$ and $H^{(h)}_{0,L}$ act on $\mathfrak{Y}^{(nh)}_{m,K} \circ \mathrm{res}_{L|K}$ via restriction along the embeddings (29) and (31), the transformation $\mathrm{r}_{\mathrm{L|K}}$ becomes $G^{(h)}_{0,L} \times H^{(h)}_{0,L}$-equivariant.

Since the functors $\mathfrak{Y}^{(h)}_{em,L}$ and $\mathfrak{Y}^{(nh)}_{m,K} \circ \mathrm{res}_{L|K}$ are representable, the transformation $\mathrm{r}_{\mathrm{L|K}}$ corresponds to the homomorphism $\mathrm{r}^*_{\mathrm{L|K}} : R^{(nh)}_{m,K} \otimes_{\breve{\mathfrak{o}}_K} \breve{\mathfrak{o}}_L \to R^{(h)}_{em,L}$ which is the image of the identity on $R^{(h)}_{em,L}$ under $\mathrm{r}_{\mathrm{L|K}}(R^{(h)}_{em,L})$. By abuse of notation we also write $\mathrm{r}_{\mathrm{L|K}}$ and $\mathrm{r}^*_{\mathrm{L|K}}$ for the induced $G^{(h)}_{0,L} \times H^{(h)}_{0,L}$-equivariant morphisms

$$\mathrm{r}_{\mathrm{L|K}} : \mathcal{Y}^{(h)}_{em,L} \to \mathcal{Y}^{(nh)}_{m,K} \times_{\breve{K}} \breve{L} \quad \text{and} \quad \mathrm{r}^*_{\mathrm{L|K}} : B^{(nh)}_{m,K} \otimes_{\breve{K}} \breve{L} \to B^{(h)}_{em,L}$$

of the associated rigid $\breve{L}$-spaces and their rings of global sections, respectively.

Proposition 5.1. *Let $L|K$ be a field extension of finite degree n and ramification index e, and let $h \geq 1$ be an integer.*

(i) *If m and m' are integers with $m' \geq m \geq 0$ then the diagram*

$$\begin{array}{ccc} \mathcal{Y}^{(h)}_{em',L} & \xrightarrow{\mathrm{r}_{\mathrm{L|K}}} & \mathcal{Y}^{(nh)}_{m',K} \times_{\breve{K}} \breve{L} \\ \downarrow & & \downarrow \\ \mathcal{Y}^{(h)}_{em,L} & \xrightarrow{\mathrm{r}_{\mathrm{L|K}}} & \mathcal{Y}^{(nh)}_{m,K} \times_{\breve{K}} \breve{L} \end{array}$$

is commutative.

(ii) *If $m \geq 0$ is an integer then the diagram*

$$\begin{array}{ccccc} \mathcal{Y}^{(h)}_{em,L} & \xrightarrow{\mathrm{r}_{\mathrm{L|K}}} & \mathcal{Y}^{(nh)}_{m,K} \times_{\breve{K}} \breve{L} & \longrightarrow & \mathcal{Y}^{(nh)}_{m,K} \\ \downarrow & & & & \downarrow \\ \mathrm{Sp}(\breve{L}_{em}) & & \longrightarrow & & \mathrm{Sp}(\breve{K}_m) \end{array}$$

is commutative and $G_{0,L}^{(h)} \times H_{0,L}^{(h)}$-equivariant. The actions of $G_{0,L}^{(h)}$ and $H_{0,L}^{(h)}$ on $\mathrm{Sp}(\breve{K}_m)$ are given by $\det_L^{-1} : G_{0,L}^{(h)} \to \mathfrak{o}_L^\times$ and $\mathrm{Nrd}_L : H_{0,L}^{(h)} \to \mathfrak{o}_L^\times$, respectively, composed with the homomorphism

$$\mathfrak{o}_L^\times \xrightarrow{N_{L|K}} \mathfrak{o}_K^\times \longrightarrow (\mathfrak{o}_K/\pi_K^m\mathfrak{o}_K)^\times \simeq \mathrm{Gal}(\breve{K}_m|\breve{K}).$$

Proof: Assertion (i) is obvious. As for (ii), the upper row is $G_{0,L}^{(h)} \times H_{0,L}^{(h)}$-equivariant by construction, and so are the vertical arrows with respect to the action exhibited in Theorem 1.4. In particular, $\mathrm{r}_{\mathrm{L|K}}^*$ restricts to an equivariant homomorphism

$$\breve{K}_m = (B_{m,K}^{(nh)})^{H_{m,K}^{(nh)}} \subseteq (B_{m,K}^{(nh)})^{H_{em,L}^{(h)}} \to (B_{em,L}^{(h)})^{H_{em,L}^{(h)}} = \breve{L}_{em},$$

giving the bottom row of the diagram. According to Theorem 1.4, the last assertion in (ii) follows from the fact that the restrictions of $\det_K : G_{0,K}^{(nh)} \to \mathfrak{o}_K^\times$ and $\mathrm{Nrd}_K : H_{0,K}^{(nh)} \to \mathfrak{o}_K^\times$ to $G_{0,L}^{(h)}$ and $H_{0,L}^{(h)}$ coincide with $\mathrm{N}_{L|K} \circ \det_L$ and $\mathrm{N}_{L|K} \circ \mathrm{Nrd}_L$, respectively. For the determinant this is well-known (cf. [1, III.9.4 Proposition 6]). Given $\alpha \in H_{0,L}^{(h)}$, choose a maximal commutative subfield L' of $D_L^{(h)}$ containing α. It is of dimension h over L, so that its image in $D_K^{(nh)}$ is a maximal commutative subfield containing K and α. We have

$$\mathrm{Nrd}_K(\alpha) = \mathrm{N}_{L'|K}(\alpha) = \mathrm{N}_{L|K} \circ \mathrm{N}_{L'|L}(\alpha) = \mathrm{N}_{L|K} \circ \mathrm{Nrd}_L(\alpha). \qquad \square$$

Note that the embeddings $H_{0,L}^{(h)} \hookrightarrow H_{0,K}^{(nh)}$ and $G_{0,L}^{(h)} \hookrightarrow G_{0,K}^{(nh)}$ extend to embeddings $H_L^{(h)} \hookrightarrow H_K^{(nh)}$ and $G_L^{(h)} \hookrightarrow G_K^{(nh)}$. Without giving the details, we remark that the morphisms $\mathrm{r}_{\mathrm{L|K}}$ extend to $G_L^{(h)} \times H_L^{(h)}$-equivariant morphisms

$$\mathrm{r}_{\mathrm{L|K}} : \underline{\mathcal{Y}}_{em,L}^{(h)} \longrightarrow \underline{\mathcal{Y}}_{m,K}^{(nh)} \times_{\breve{K}} \breve{L}$$

of the corresponding Rapoport-Zink spaces. These give rise to a continuous $G_L^{(h)} \times H_L^{(h)}$-equivariant homomorphism $\mathrm{r}_{\mathrm{L|K}}^* : \mathbb{B}_{\infty,K}^{(nh)} \to \mathbb{B}_{\infty,L}^{(h)}$ of topological $\breve{K}$-algebras, as well as to an $H_L^{(h)}$-equivariant morphism

$$\mathrm{r}_{\mathrm{L|K}} : \mathbb{P}_{\breve{L}}^{h-1} \to \mathbb{P}_{\breve{K}}^{nh-1} \times_{\breve{K}} \breve{L}.$$

As for the Drinfeld tower, given a field extension $L|K$ of finite degree n, an integer $h \geq 1$ and starting from a K-linear embedding $D_L^{(h)} \hookrightarrow D_K^{(nh)}$, Drinfeld constructed in [15, §3], a closed embedding

$$\mathrm{i}_{\mathrm{L|K}} : \mathfrak{X}_{0,L}^{(h)} \longrightarrow \mathfrak{X}_{0,K}^{(nh)} \times_{\breve{\mathfrak{o}}_K} \breve{\mathfrak{o}}_L. \tag{32}$$

Its construction relies on the fact that $\mathfrak{o}_{D_K^{(nh)}} \otimes_{\mathfrak{o}_{D_L^{(h)}}} \mathbb{G}_L^{(h)}$ is a special formal $\mathfrak{o}_{D_K^{(nh)}}$-module of height $(nh)^2$ over k^s, hence is isomorphic to $\mathbb{G}_K^{(nh)}$ (cf. [15, §2.1]). Any such isomorphism induces an embedding

$$\mathrm{M}_h(L) \simeq \mathrm{End}_{\mathfrak{o}_{D_L^{(h)}}}(\mathbb{G}_L^{(h)}) \otimes_{\mathfrak{o}_L} L \hookrightarrow \mathrm{End}_{\mathfrak{o}_{D_K^{(nh)}}}(\mathbb{G}_K^{(nh)}) \otimes_{\mathfrak{o}_K} K \simeq \mathrm{M}_{nh}(K) \tag{33}$$

of K-algebras, giving rise to an embedding of the subgroup of $G_L^{(h)}$ consisting of elements with determinant in $\mathfrak{o}_L^\times$ into the subgroup of $G_K^{(nh)}$ consisting of elements with determinant in $\mathfrak{o}_K^\times$. It follows from its functorial construction that the morphism $\mathrm{i_{L|K}}$ is $G_{0,L}^{(h)} \times H_{0,L}^{(h)}$-equivariant.

We denote by $\mathrm{i_{L|K}} : \mathcal{X}_{0,L}^{(h)} \to \mathcal{X}_{0,K}^{(nh)} \times_{\breve{K}} \breve{L}$ the induced morphism of rigid analytic $\breve{L}$-varieties. According to [15, §3] it induces closed equivariant embeddings $\mathrm{i_{L|K}} : \mathcal{X}_{em,L}^{(h)} \to \mathcal{X}_{m,K}^{(nh)}$ for any integer $m \geq 0$, where $e = e_{L|K}$ denotes the ramification index of the extension $L|K$. We denote by $\mathrm{i}^*_{\mathrm{L|K}} : A_{em,L}^{(h)} \to A_{m,K}^{(nh)}$ the induced continuous equivariant homomorphisms of $\breve{K}$-Fréchet algebras.

Proposition 5.1 has an exact analog in this situation which we refrain from repeating.

Again, the morphims $\mathrm{i_{L|K}}$ extend to $G_L^{(h)} \times H_L^{(h)}$-equivariant morphisms $\mathrm{i_{L|K}} : \underline{\mathcal{X}}_{em,L}^{(h)} \to \underline{\mathcal{X}}_{m,K}^{(nh)} \times_{\breve{K}} \breve{L}$ between the corresponding Rapoport-Zink spaces and give rise to a continuous $G_L^{(h)} \times H_L^{(h)}$-equivariant homomorphism

$$\mathrm{i}^*_{\mathrm{L|K}} : \mathbb{A}_{\infty,K}^{(nh)} \longrightarrow \mathbb{A}_{\infty,L}^{(h)}$$

of topological $\breve{K}$-algebras. We also denote by $\mathrm{i_{L|K}}$ the induced $G_L^{(h)}$-equivariant morphism

$$\mathrm{i_{L|K}} : \mathcal{X}_{0,L}^{(h)} \simeq \underline{\mathcal{X}}_{0,L}^{(h)}/H_L^{(h)} \longrightarrow \mathcal{X}_{0,K}^{(nh)} \simeq \underline{\mathcal{X}}_{0,K}^{(nh)}/H_K^{(nh)}.$$

We shall now study the behavior of Lubin-Tate and Drinfeld bundles under pull back along $\mathrm{i_{L|K}}$ and $\mathrm{r_{L|K}}$, respectively. If Γ is a locally profinite group and if Γ' is a closed subgroup then we denote by $\mathrm{res} = \mathrm{res}^{\Gamma}_{\Gamma'} : \mathrm{Rep}^{\infty}_{\breve{K}}(\Gamma) \to \mathrm{Rep}^{\infty}_{\breve{K}}(\Gamma')$ the restriction functor.

Theorem 5.2. *Let $L|K$ be a field extension of finite degree n, and let $h \geq 1$ be an integer. The two diagrams*

$$\begin{array}{ccc} \mathrm{Rep}^{\infty}_{\breve{K}}(G_{0,K}^{(nh)}) & \xrightarrow{\mathcal{M}} & \mathcal{B}_{\mathcal{X}_{0,K}^{(nh)}}(G_{0,K}^{(nh)}) \\ {\scriptstyle \mathrm{res}\circ(\breve{L}\otimes_{\breve{K}}(\,\cdot\,))}\downarrow & & \downarrow{\scriptstyle \mathrm{i}^*_{\mathrm{L|K}}} \\ \mathrm{Rep}^{\infty}_{\breve{L}}(G_{0,L}^{(h)}) & \xrightarrow{\mathcal{M}} & \mathcal{B}_{\mathcal{X}_{0,L}^{(h)}}(G_{0,L}^{(h)}) \end{array}$$

and

$$\begin{array}{ccc} \mathrm{Rep}^{\infty}_{\breve{K}}(H_{0,K}^{(nh)}) & \xrightarrow{\mathcal{N}} & \mathcal{B}_{\mathcal{Y}_{0,K}^{(nh)}}(H_{0,K}^{(nh)}) \\ {\scriptstyle \mathrm{res}\circ(\breve{L}\otimes_{\breve{K}}(\,\cdot\,))}\downarrow & & \downarrow{\scriptstyle \mathrm{r}^*_{\mathrm{L|K}}} \\ \mathrm{Rep}^{\infty}_{\breve{L}}(H_{0,L}^{(h)}) & \xrightarrow{\mathcal{N}} & \mathcal{B}_{\mathcal{Y}_{0,L}^{(h)}}(H_{0,L}^{(h)}) \end{array}$$

are commutative. In particular, $\mathrm{i}^*_{\mathrm{L|K}}(\mathcal{M}(V))$ and $\mathrm{r}^*_{\mathrm{L|K}}(\mathcal{N}(W))$ are Lubin-Tate and Drinfeld bundles on $\mathcal{X}_{0,L}^{(h)}$ and $\mathcal{Y}_{0,L}^{(h)}$ whenever V and W are finite dimensional smooth representations of $H_{0,K}^{(nh)}$ and $G_{0,K}^{(nh)}$ over $\breve{K}$, respectively. In this *case there are natural isomorphisms*

$$\mathbb{D}_{\mathrm{LT}}(\mathrm{i}^*_{\mathrm{L|K}}(\mathcal{M}(V))) \simeq \mathrm{r}^*_{\mathrm{L|K}}(\mathbb{D}_{\mathrm{LT}}(\mathcal{M}(V))) \quad \textit{and} \tag{34}$$

$$\mathbb{D}_{\mathrm{Dr}}(\mathrm{r}^*_{\mathrm{L|K}}(\mathcal{N}(W))) \simeq \mathrm{i}^*_{\mathrm{L|K}}(\mathbb{D}_{\mathrm{Dr}}(\mathcal{N}(W))) \tag{35}$$

in $\mathcal{B}_{\mathcal{Y}^{(h)}_{0,L}}(H^{(h)}_{0,L})$ *and* $\mathcal{B}_{\mathcal{X}^{(h)}_{0,L}}(G^{(h)}_{0,L})$*, respectively.*

Proof: The commutativity of the two diagrams is clear, so that the second assertion is a consequence of Theorem 3.8. In the proof of the latter we saw that there is a natural $H^{(nh)}_{0,K}$-equivariant isomorphism

$$\mathbb{D}_{\mathrm{LT}}(\mathcal{M}(V)) = (C^{(nh)}_{m,K} \otimes_{\breve{K}} V)^{G^{(nh)}_{0,K}} \simeq (B^{(nh)}_{m,K} \otimes_{\breve{K}} V)^{G^{(nh)}_{0,K}} \tag{36}$$

for any object V of $\mathrm{Rep}^{\infty}_{\breve{K}}(G^{(nh)}_{0,K})$ if the integer m is chosen so that $G^{(nh)}_{m,K}$ acts trivially on V. Further, the natural map

$$B^{(nh)}_{m,K} \otimes_{B^{(nh)}_{0,K}} (B^{(nh)}_{m,K} \otimes_{\breve{K}} V)^{G^{(nh)}_{0,K}} \longrightarrow B^{(nh)}_{m,K} \otimes_{\breve{K}} V$$

is a $G^{(nh)}_{0,K}$-equivariant isomorphism. Since the $B^{(nh)}_{0,K}$-module $\mathbb{D}_{\mathrm{LT}}(\mathcal{M}(V))$ is projective, passage to $G^{(h)}_{0,L}$-invariants gives

$$(B^{(nh)}_{m,K} \otimes_{\breve{K}} V)^{G^{(h)}_{0,L}} \simeq (B^{(nh)}_{m,K})^{G^{(h)}_{0,L}} \otimes_{B^{(nh)}_{0,K}} (B^{(nh)}_{m,K} \otimes_{\breve{K}} V)^{G^{(nh)}_{0,K}}.$$

Tensoring with $B^{(nh)}_{m,K}$ over $(B^{(nh)}_{m,K})^{G^{(h)}_{0,L}}$ shows that the natural $G^{(h)}_{0,L}$-equivariant homomorphism

$$B^{(nh)}_{m,K} \otimes_{(B^{(nh)}_{m,K})^{G^{(h)}_{0,L}}} (B^{(nh)}_{m,K} \otimes_{\breve{K}} V)^{G^{(h)}_{0,L}} \longrightarrow B^{(nh)}_{m,K} \otimes_{\breve{K}} V$$

is bijective.

Note that $(B^{(nh)}_{m,K})^{G^{(h)}_{0,L}}$ is the ring of global sections of $\mathcal{Y}^{(nh)}_{m,K}/G^{(h)}_{0,L}$. Since the covering $\mathcal{Y}^{(nh)}_{m,K} \to \mathcal{Y}^{(nh)}_{0,K}$ is finite étale and Galois, so is the covering $\mathcal{Y}^{(nh)}_{m,K} \to \mathcal{Y}^{(nh)}_{m,K}/G^{(h)}_{0,L}$. In particular, the homomorphism $(B^{(nh)}_{m,K})^{G^{(h)}_{0,L}} \to B^{(nh)}_{m,K}$ is faithfully flat (cf. Theorem 1.2 and Proposition A.5). It follows that the $(B^{(nh)}_{m,K})^{G^{(h)}_{0,L}}$-module $(B^{(nh)}_{m,K} \otimes_{\breve{K}} V)^{G^{(h)}_{0,L}}$ is projective. Thus, there are $H^{(h)}_{0,L}$-equivariant isomorphisms

$$(B^{(h)}_{em,L} \otimes_{\breve{K}} V)^{G^{(h)}_{0,L}} \simeq B^{(h)}_{0,L} \otimes_{B^{(nh)}_{0,K}} (B^{(nh)}_{m,K} \otimes_{\breve{K}} V)^{G^{(nh)}_{0,K}}. \tag{37}$$

In particular, $\mathrm{i}^*_{\mathrm{L|K}}(\mathcal{M}(V))$ is trivialized by $B^{(h)}_{em,L}$. Combining the above with the natural isomorphisms

$$\mathbb{D}_{\mathrm{LT}}(\mathrm{i}^*_{\mathrm{L|K}}(\mathcal{M}(V))) \simeq (B^{(h)}_{em,L} \otimes_{\breve{K}} V)^{G^{(h)}_{0,L}}$$

and (36), this proves (34). The functoriality assertion in (35) can be proved analogously. $\square$

For vector bundles coming from smooth representations, the functoriality properties of Theorem 5.2 extend to the equivalence in Theorem 4.4 involving the full groups $G^{(nh)}_{K}$ and $H^{(nh)}_{K}$.

Theorem 5.3. *Let $L|K$ be a field extension of finite degree n and let $h \geq 1$ be an integer. If ρ is an object of $\mathrm{Rep}^{\infty}_{\breve{K}}(H_K^{(nh)})$ then the $G_L^{(h)}$-equivariant vector bundle $\mathrm{i}^*_{\mathrm{L|K}}(\mathbb{D}_{\mathrm{Dr}}(\rho))$ on $\mathcal{X}_{0,L}^{(h)}$ is Lubin-Tate. In fact, there is a natural $G_L^{(h)}$-equivariant isomorphism*

$$\mathrm{i}^*_{\mathrm{L|K}}(\mathbb{D}_{\mathrm{Dr}}(\rho)) \simeq \mathbb{D}_{\mathrm{Dr}}(\mathrm{res}^{H_K^{(nh)}}_{H_L^{(h)}}(\breve{L} \otimes_{\breve{K}} \rho)). \tag{38}$$

Proof: As in the proof of Theorem 3.8 we see that there is an integer $m \geq 0$ and $G_{0,K}^{(nh)}$-equivariant isomorphisms

$$\mathbb{D}_{\mathrm{Dr}}(\rho) \simeq (\mathbb{A}_{m,K}^{(nh)} \otimes_{\breve{K}} \rho)^{H_K^{(nh)}} \simeq (A_{m,K}^{(nh)} \otimes_{\breve{K}} \rho)^{H_{0,K}^{(nh)}}.$$

Thus, we may argue as before. □

A Results from rigid geometry

We will prove several results from rigid geometry for which we could not find suitable references. Throughout the appendix, let F be a field which is complete with respect to a nontrivial nonarchimedean valuation.

Lemma A.1. *If Z is a normal connected rigid analytic F-variety then the ring of global sections of Z is an integrally closed integral domain.*

Proof: It follows from [10, Lemma 2.1.4], that $S := \mathcal{O}(Z)$ is an integral domain. Further, Z admits an affinoid covering $(Z_i)_{i\in I}$ such that all affinoid spaces Z_i are normal and connected. By [10, Lemma 2.1.4] and [3, V.1.5 Corollaire 3], each of the rings $S_i := \mathcal{O}(Z_i)$ is an integrally closed integral domain. The sheaf axioms imply that also S is integrally closed. □

Let Z be a rigid analytic F-variety. We call *vector bundle of finite rank on Z* a coherent locally free $\mathcal{O}_Z$-module $\mathcal{M}$ such that

$$\sup_{z\in Z}\{\mathrm{rk}_{\mathcal{O}_{Z,z}}\mathcal{M}_z\} < \infty. \tag{39}$$

If Z has only finitely many connected components then the global finiteness condition (39) is satisfied by any coherent locally free $\mathcal{O}_Z$-module.

Proposition A.2. *Let Z be a quasi-Stein rigid analytic F-variety such that $\sup_{z\in Z}\{\dim(\mathcal{O}_{Z,z})\} < \infty$. The global section functor is an equivalence between the category of vector bundles of finite rank on Z and the category of finitely generated projective $\mathcal{O}(Z)$-modules.*

Proof: Let $(Z_i)_{i\in\mathbb{N}}$ be an affinoid covering exhibiting Z as a quasi-Stein space. It follows from Theorem B (cf. [29, Satz 2.4]) that the assignment $M \mapsto \tilde{M}$, with $\tilde{M}(Z_i) := \mathcal{O}(Z_i) \otimes_{\mathcal{O}(Z)} M$, is quasi-inverse to the global section functor, considered on the larger category of coherent module sheaves on Z. A proof of this fact in the more general setting of possibly noncommutative analogs of $\mathcal{O}(Z)$ can be found in [40, Section 3]. Any finitely generated projective $\mathcal{O}(Z)$-module M is finitely presented and hence is contained in the essential image of the global section functor (cf. [40, Corollary 3.4]). Evidently, the sheaf $\tilde{M}$ is locally free of finite rank.

Conversely, if $\mathcal{M}$ is a vector bundle of finite rank then $\mathcal{M}(Z)$ is a finitely generated projective $\mathcal{O}(Z)$-module. Indeed, by [3, VIII.1.3 Proposition 8] and our assumption, there is an index i_0 such that $\dim(\mathcal{O}(Z_i)) = \dim(\mathcal{O}(Z_{i_0}))$ for all indices $i \geq i_0$. The claim can now be proved along the lines of [25, p. 84], proof of Théorème 1. It is here that we need the global bound (39). $\square$

Let Z be a rigid analytic F-variety endowed with the left action of a group Γ. Recall that an $\mathcal{O}_Z$-module $\mathcal{M}$ is called *left* Γ*-equivariant* if there is a family of isomorphisms $c_\gamma : (\gamma^{-1})^*(\mathcal{M}) \to \mathcal{M}$, $\gamma \in \Gamma$, satisfying the relations $c_1 = \mathrm{id}_{\mathcal{M}}$ and $c_{\gamma_2} \circ (\gamma_2^{-1})^*(c_{\gamma_1}) = c_{\gamma_2\gamma_1}$ for any two elements $\gamma_1, \gamma_2 \in \Gamma$.

By spelling out the definition of an equivariant sheaf, the following corollary is an immediate consequence of Proposition A.2.

Corollary A.3. *Let Z be a quasi-Stein rigid analytic F-variety endowed with the action of a group Γ. Assuming $\sup_{z\in Z}\{\dim(\mathcal{O}_{Z,z})\} < \infty$, the global section functor is an equivalence between the category of Γ-equivariant vector bundles of finite rank on Z and the category of finitely generated projective $\mathcal{O}(Z)$-modules carrying a semilinear action of Γ.* $\square$

By reducing to a local situation, the following theorem can be proved using general facts on étale Galois descent for schemes (cf. [7, Example 6.2.B]).

Theorem A.4. *If $f : Z \to Z'$ is a finite étale Galois morphism of rigid analytic F-varieties, and if Γ denotes the corresponding Galois group, then the inverse image functor f^* is an equivalence between the category of coherent $\mathcal{O}_{Z'}$-modules and the category of Γ-equivariant coherent $\mathcal{O}_Z$-modules. A quasi-inverse is given by the functor sending a Γ-equivariant coherent $\mathcal{O}_Z$-module $\mathcal{M}$ to $f_*(\mathcal{M})^\Gamma$. Locally free sheaves correspond to locally free sheaves of the same rank.* $\square$

Proposition A.5. *Let $f : Z \to Z'$ be a finite flat morphism of quasi-Stein rigid analytic F-varieties and set $R := \mathcal{O}(Z)$ and $S := \mathcal{O}(Z')$. Assuming Z to have only finitely many connected components and $\sup_{z'\in Z'}\{\dim(\mathcal{O}_{Z',z'})\} < \infty$, the S-module R is finitely generated and projective. If, moreover, Z is non-empty, and if Z' is normal and connected, then R is faithfully flat over S.*

Proof: If Z has only finitely many connected components then the $\mathcal{O}_{Z'}$-module $f_*\mathcal{O}_Z$ is locally free of finite rank in the strong sense of (39). Thus, under the assumptions on Z', R is a finitely generated projective S-module according to Proposition A.2.

It follows from Lemma A.1 that S is an integral domain. If Z is non-empty then R is nonzero, and the flat homomorphism $S \to R$ is injective. Since it is also finite, it follows from [3, II.2.5 Corollaire 4 and V.2.1 Théorème 1], that R is faithfully flat over S. $\square$

Proposition A.6. *Let $f : Z \to Z'$ be a finite flat morphism of rigid analytic F-varieties. If Z is non-empty and if Z' is connected then f is surjective.*

Proof: The image of f is a Zariski closed subset of Z' by [6, 9.6.3 Proposition 3]. If $U \subseteq Z'$ is an affinoid subdomain then the restriction of f to $f^{-1}(U)$ is a finite flat morphism of affinoid spaces. According to [32, Theorem I.2.12], the subset $f(f^{-1}(U))$ of U is Zariski open. It follows that $f(Z)$ is Zariski open in Z'. Since the image of f is non-empty the connectedness of Z' implies that $f(Z) = Z'$. $\square$

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