ERRATUM:

P-ALCOVES AND NONEMPTINESS OF AFFINE DELIGNE-LUSZTIG VARIETIES

P-ALCÔVES ET VACUITÉ DE VARIÉTÉS DE DELIGNE-LUSZTIG AFFINES

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In this erratum, we would like to rectify two errors in our paper [2]. In both cases, the mistake has no consequences for other parts of the paper; only the statements of the corrected propositions below are used.

Reduction to adjoint groups. Proposition 2.2.1 does not hold as stated, and should be replaced by the following:

Proposition 0.0.1. *Assume that* $\operatorname{char} \mathbb{k}$ *does not divide the order of* $\pi_1(\mathbf{G}_{\rm ad})$. *Let* $\lambda \in \pi_0(\text{Flag}) = \pi_1(\mathbf{G})_\Gamma$, *denote by* $\lambda_{\rm ad}$ *its image under the map* $\pi_0(\text{Flag}) \to \pi_0(\text{Flag}_{\rm ad})$, *and denote by* Flag_λ *and* $\text{Flag}_{{\rm ad},\lambda_{\rm ad}}$ *the corresponding connected components. Then the projection* $\mathbf{G} \to \mathbf{G}_{\rm ad}$ *induces an isomorphism*

$$\text{Flag}_\lambda \xrightarrow{\cong} \text{Flag}_{{\rm ad},\lambda_{\rm ad}}.$$

If the map $\pi_0(\text{Flag}) \to \pi_0(\text{Flag}_{\rm ad})$ *is injective, then the homomorphism* $\mathbf{G} \to \mathbf{G}_{\rm ad}$ *induces an immersion*

$$\text{Flag} \to \text{Flag}_{\rm ad}.$$

The proof given in the paper proves the above statement. The problem with the original statement is that the map $\pi_0(\text{Flag}) \to \pi_0(\text{Flag}_{\rm ad})$ is *not injective in general*; this was erroneously claimed in part (2) of the original statement, and implicitly used in part (1).

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Assume that $\mathbf{G}$ is semisimple. In this case the map $\pi_0(\mathrm{Flag}) \to \pi_0(\mathrm{Flag}_{\mathrm{ad}})$ is injective if and only if the coinvariants $X_*(T)_\Gamma$ are torsion-free (which is true for instance, if G is of adjoint type or simply connected, see [1] 4.4.16). In fact, using the notation of [3], we can identify

$$\pi_0(\mathrm{Flag}) = \pi_1(G)_\Gamma = X^*(\widehat{Z}(G)^\Gamma) = X_*(T)_\Gamma / X_*(T_{\mathrm{sc}})$$

(and likewise for G_{ad}), see loc. cit., page 196. In view of the commutative diagram

$$\begin{array}{ccccc} X_*(T_{\mathrm{sc}})_\Gamma & \hookrightarrow & X_*(T)_\Gamma & \longrightarrow & X_*(T)_\Gamma \otimes_{\mathbb{Z}} \mathbb{Q} \\ \downarrow {\scriptstyle =} & & \downarrow & & \downarrow {\scriptstyle \cong} \\ X_*(T_{\mathrm{sc}})_\Gamma & \hookrightarrow & X_*(T_{\mathrm{ad}})_\Gamma & \hookrightarrow & X_*(T_{\mathrm{ad}})_\Gamma \otimes_{\mathbb{Z}} \mathbb{Q} \end{array}$$

we see that

$$\ker(\pi_0(\mathrm{Flag}) \to \pi_0(\mathrm{Flag}_{\mathrm{ad}})) \cong X_*(T)_{\Gamma,\mathrm{tors}},$$

the torsion subgroup of $X_*(T)_\Gamma$.

Some properties on Newton points. In Proposition 3.5.1, the assumption that $[b]$ be basic is missing. The correct statement is

Proposition 0.0.2. *Let $[b]$ be a basic σ-conjugacy class in $\mathbf{G}(\mathbb{L})$ and $J \subset S$ with $\delta(J) = J$. Then $[b] \cap \mathbf{M}_J(\mathbb{L})$ contains at most one σ-conjugacy class of $\mathbf{M}_J(\mathbb{L})$.*

This statement is justified by the proof of the proposition given in [2]. The problem in the non-basic case is that in line 3 we can only really conclude that $\bar{\nu}_x = \bar{\nu}_{x'}$, i.e., that the *dominant* Newton vectors of x and x' coincide. However, in the sequel of the proof we use the stronger statement that $\nu_x = \nu_{x'}$. If x lies in a *basic* σ-conjugacy class, then $\nu_x = \bar{\nu}_x$ since $\bar{\nu}_x$ is central.

It is easy to give counterexamples to the statement for non-basic b, e.g., take $\mathbf{G} = GL_2$, $J = \emptyset$, i.e., $\mathbf{M} = \mathbf{M}_J$ is the diagonal torus. Then the diagonal matrices with entries $(\epsilon, 1)$, and $(1, \epsilon)$, resp., are σ-conjugate in $\mathbf{G}(L)$, but not in $\mathbf{M}(L)$.

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References

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